LIV. The Electric Origin of Rigidity and Consequences.
By William Sutherland.*

In "A Kinetic Theory of Solids" (Phil. Mag. [5] xxxii.) it was stated that a certain formula almost demonstrated the kinetic origin of rigidity. This formula for the rigidity \( n \) of a metal of melting-point \( T \) at temperature \( \theta \), the rigidity at absolute zero being \( N \), was

\[
\frac{n}{N} = 1 - \left( \frac{\theta}{T} \right)^2
\]  

Further inquiry on the lines of "The Electric Origin of Molecular Attraction" (Phil. Mag. [6] iv.) has led me to amend the above statement to the following form:—The variation of rigidity with temperature is a simple kinetic phenomenon. It will now be shown that rigidity at absolute zero is a purely electrostatic affair, and certain immediate consequences of this fact will be worked out according to the following table of contents:—

1. The Electric Origin of Rigidity.
2. The consequent nature of Atomic Vibration.
3. The division of Molecular Electric Energy into Electrostatic and Electrokinetic as shown by the temperature variation of rigidity in metals.

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6. The Neutron Structure of the Ether used for calculation of its Density and Rigidity, with deduction of the Velocity of Light.

1. The Electric Origin of Rigidity.

The simplest way of showing the electrostatic origin of rigidity is the following. Equation (16) of "A Kinetic Theory of Solids" gives us that

\[ 21NbM/JcM = 4.6 \]

(2)

where \( b \) is the coefficient of linear expansion, \( M \) the atomic mass, that of hydrogen being 1, \( c \) the specific heat, \( \rho \) the density, and \( J \) the mechanical equivalent of heat. By means of the experimental data this was shown to be approximately true for twelve metals, while Zn and Cd give a value almost double the 4.6. In my various papers on molecular force the characteristic parameter of molecular attraction for many substances is tabulated under the heading \( M^{2/l} \), in which \( M \) is the molecular mass and \( l \) is \( 2/3 \) of the virial parameter for unit mass of the substance. With the inverse fourth power for the law of molecular attraction \( lp \) gives the potential energy of unit mass of the substance at density \( \rho \). But as molecular attraction is not directly dependent on molecular mass, it is better to treat \( M^{2/l} \) as a single parameter independent of \( M \). Now in "Further Studies on Molecular Force" (Phil. Mag. [5] xxxix.) equation (7) is

\[ M^{2/l} = 61J(M/\rho)/b \]

(3)

By the law of Dulong and Petit \( cM \) has the mean value 6.4, thus (2) and (3) give

\[ N = 2.3 \frac{M^{2/l}}{(M/\rho)^2} \]

(4)

Now if \( m \) is the actual mass of a molecule it may replace \( M \) in (4). But in "The Electric Origin of Molecular Attraction" it was shown that \( m^{2/l} \) stands for \( e^2s^2 \), where \( e \) is the electron charge and \( s \) is the distance between the positive and negative electrons forming the electric doublets which cause molecular attraction. These electrons can be denoted by \( \# \) and \( \$ \).

If the dielectric capacity of the medium is \( K \) instead of 1, \( m^{2/l} = e^2s^2/K \). In molecules containing more than one atom \( \# \) and \( \$ \) are the Helmholtz valency charges and the ionic charges of electrolytes. But in metals each monatomic
molecule must contain \( \mathfrak{g} \) and \( \mathfrak{h} \) to form the doublet which gives cohesion. This is the most important characteristic of the molecules in metals, that each atom contains in itself both \( \mathfrak{g} \) and \( \mathfrak{h} \). This conception is fundamental in the present theory, especially in section 5.

Equation (4) becomes

\[
N = \frac{2 \cdot 3 \varepsilon^2 s^2}{K (m/\rho)^2} \cdot \cdot \cdot \cdot (5)
\]

This shows the electric origin of rigidity, but it needs interpreting. Consider the electric doublets in a metal at absolute zero so arranged that their charges form alternate plane laminae of positive and negative electricity of surface-density \( \sigma \). The average electric force between the laminae must be \( 4\pi \sigma /K \); and according to Maxwell’s view the energy in unit volume of the dielectric is \( \frac{1}{2} \sigma \cdot 4\pi \sigma /K \). To make our laminar distribution correspond with the charges of the electric doublet we must have \( \sigma = \varepsilon/(m/\rho)^{\frac{3}{2}} \); and, moreover, to give the properties of a homogeneous body we should use three laminar distributions at right angles to one another, such that each would give one-third of the energy due to the doublets in the actual body which they represent. Thus, then, the electrostatic energy per unit volume due to the components of the doublets in any one direction can be written

\[
\frac{2\pi}{3K} \cdot \varepsilon^2 (m/\rho)^{\frac{3}{2}} = \frac{2 \cdot 1 \varepsilon^2 (m/\rho)^{\frac{3}{2}}}{K (m/\rho)^2} \cdot \cdot (6)
\]

This becomes identical with (5) if \( s = (m/\rho)^{\frac{3}{2}} \), a result which would make it appear that in the monatomic molecules of metals the two electrons \( \mathfrak{g} \) and \( \mathfrak{h} \) are on the surface of the atom at the ends of a diameter. With this result it is possible, by means of the values for \( M^2 \) for the metals given in Table xxix. of “Further Studies” and by the equation \( m^2 = \varepsilon^2 s^2 /K \), to obtain values of \( K \) the mean dielectric capacity of the metals, but we cannot at present follow this side track. We must now show how the rigidity at absolute zero and the electrostatic energy of the doublets resolved in any one direction are identical.

Consider two electrons \( \mathfrak{g} \) and \( \mathfrak{h} \) of amount \( e \) at distance \( r \) apart in the positions A and B. Subject them to a small shear of amount \( u \) indicated by the displacements AC and BD each equal to \( u/2 \) at right angles to AB. The electrical work done is \( d(e^2 /rK) = -\varepsilon^2 dr /r^2 K \). But \( dr = u^2 r^2/2 \), and therefore the work done
is equal to half the square of the shear multiplied by the original electric energy $e^2/\rho K$. Thus the rigidity of a pair of electrons is their electrostatic energy. To obtain a simple dynamical representation of a metal we may use the fiction of three laminar distributions, and then, by the definition of rigidity and Maxwell's expression $2\pi \sigma^2/K$ for the energy in unit volume of a dielectric, we get by a similar argument to that just used for a pair of electrons the result

$$N = \frac{2\pi}{3K} \cdot \frac{e^2 (m/\rho)^{\frac{3}{2}}}{(m/\rho)^{\frac{3}{2}}} \ldots \ldots \ldots (7)$$

This demonstrates the electric origin of rigidity at absolute zero. At higher temperatures kinetic effects have to be taken into account as well as the static. Rigidity is of electrostatic origin, but its variation with temperature is a simple kinetic phenomenon. I find (Physical Review, x. 1900) that R. A. Fessenden, in America, for some time has been advocating the electric origin of cohesion and rigidity, though in spite of considerable insight and imagination he has not, to my knowledge, formulated his ideas with sufficient precision for a working physical theory.

Reinganum, who has investigated the electric origin of cohesion (Physikalische Zeitschrift, 1900, Ann. der Ph. [4] x. 1903), has, in the second of these papers, considered the tensile strength of metals in a general way without special attention to rigidity.


In section 6 of "The Cause of the Structure of Spectra" (Phil. Mag. [6] ii.) the mechanical period of vibration of an atom of metal was calculated in the following way. The rigidity of the metal at absolute zero must be the rigidity of its monatomic molecule. The velocity of propagation of a shearing stress through the atom must be $(N/\rho)^{\frac{1}{2}}$. But as $N = 2\pi \sigma^2/3K$, this takes the form $\sigma (2\pi/3\rho K)^{\frac{1}{2}}$. Since the linear dimension of the atom is $(m/\rho)^{\frac{1}{2}}$ or $2a$, the period of vibration of the atom is

$$2a/\sigma (2\pi/3\rho K)^{\frac{1}{2}} \ldots \ldots \ldots (8)$$

On account of the electric origin of rigidity, the period of mechanical vibration is expressed in terms of the electric properties of the atom. Accordingly, without the intervention of the idea of rigidity, we can calculate and interpret this vibration as an electrical-mechanical analogue of the vibration of an ordinary magnet. The inertia of the atom
corresponds to the inertia of the magnet, and the electric doublet to the two magnetic poles. Consider our doublet in a spherical atom of mass \( m \) and radius \( a \), and suppose this atom vibrating in the field of electric force due to all the other atoms, the strength of this field being \( \frac{4\pi \sigma}{K} \). The period is

\[
2\pi \left( \frac{\text{moment of inertia}}{\text{moment of force}} \right)^{\frac{1}{2}} = 2\pi a \left( \frac{m^2}{4 \pi \sigma/K} \right)^{\frac{1}{2}} = \frac{2\pi a}{(30 \cdot \frac{3}{2} \pi \frac{a^2}{m} \sigma^2 s/Ka)^{\frac{1}{2}}}. 
\]

But as we must suppose only one-third of the energy per unit volume associated with the field of force in any one direction, we must in the last expression replace \( \sigma^2 \) by \( \sigma^2/3 \), and we get on numerical reduction with \( s = 2a \),

\[
2a/\sigma(2/\rho K)^{\frac{1}{2}}. \quad \ldots \ldots (9)
\]

The expression (8) on numerical simplification reduces to the same as (9) except that \( 2 \cdot 1 \) for \( 2\pi/3 \) replaces \( 2\cdot0 \). Thus we see that what I called the mechanical period of vibration of an atom in my paper on Spectra, is capable of the simple electrical interpretation which it must have if rigidity at absolute zero is of electric origin. If this fundamental period is the same at all temperatures, then, as regards actions so rapid as the vibrations of the atom, the whole electric energy must act as if it retained the electrostatic form and the same value as at absolute zero. Note that we have found the electric doublet forcing the material atom to vibrate. For other actions we shall find that with rising temperature more and more of the original stock of electrostatic energy at absolute zero appears in the form of electrokineetic energy.

The double interpretation of atomic vibrations as mechanical and electrical is another instance of a certain duality which must accompany the attempt to treat the properties of matter and electricity as identical. Maxwell's electromagnetic theory has led to the current fruitful speculations on the electric constitution of matter. The older mechanical theory of light, of which Kelvin is the leading living cultivator, has as its logical goal a material theory of electricity. Both roads lead ultimately to the same junction, and duality such as that just discussed must be the rule. The connexion between the vibration of atoms here discussed and the translatory vibrations of the monatomic metal molecules at their melting-point, has been pointed out in "The Cause of the Structure of Spectra."
3. The Division of Molecular Electric Energy into Electrostatic and Electrokinetic as shown by the Temperature Variation of Rigidity in Metals.

The theory of rigidity leads as follows to a fundamental principle concerning the partition of electric energy. Let us denote the electrostatic energy associated with matter per unit volume in a metal at absolute zero by \( W^2 \), and assume that \( n \) the rigidity at \( \theta \) represents the electrostatic energy at that temperature, denoted by \( w^2 \), then by (1)

\[
\frac{w^2}{W^2} = 1 - \frac{\theta^2}{T^2}.
\]

Now as a metal is heated up from zero to \( \theta \) its atoms have an increasing amplitude of motion which at \( \theta \) is proportional to \( bT \) the expansion; and it may be assumed that, just as in harmonic motion mean velocity is proportional to mean amplitude, so the angular velocity of atoms of metal at \( \theta \) is proportional to \( bT \) or equal to \( c\theta \), where \( c \) is a constant for any given metal. If, then, the difference between \( W^2 \) and \( w^2 \) exists as rotatory energy \( C\theta^2 \), we have the equation

\[
W^2 - w^2 = C\theta^2.
\]

But at the melting-point rigidity vanishes and \( w^2 = 0 \), so that when \( \theta = T \) we find \( W^2 = CT^2 \), and thus

\[
\frac{w^2}{W^2} = 1 - \frac{\theta^2}{T^2}.
\]

The above, then, are the assumptions by which I propose to give a dynamical meaning to (1). The fraction \( \theta^2/T^2 \) represents that fraction of the original electrostatic energy at absolute zero which is converted into electrokinetic energy on raising the temperature to \( \theta \). The combined existence of electrostatic and electrokinetic energies in the same pair of electrons may be conceived by imagining the electrons to travel over the surface of the atom in curves similar to screws having a small pitch so that the motion parallel to the axis of the screw is negligible in comparison with that round it. The average distance along the axis of the screw between the two electrons is the distance \( s \) which gives the pair of electrons the electric moment \( es \). The electrokinetic energy of motion round this axis and the electrostatic energy due to separation of the charges along the axis, may both be regarded as a store of directed energy of total amount \( W^2 \) per unit volume. In this way we conceive a possible source of phenomena in which the total energy \( W^2 \) predominates, and of
others in which the parts \( w^2 \) and \( W^2/\Theta^2 \) separately predominate. Thus when a metal melts, \( w^2 \) and the rigidity \( n \) vanish, because the electric axis in each molecule changes direction so quickly that the mean electric moment vanishes. For actions more rapid, small portions of a melted metal might exhibit high rigidity, all depending on Maxwell's time of relaxation. In the next section the electrokinetic energy \( W^2/\Theta^2 \) will play an important part in accounting for the mechanism of electric and thermal conduction in metals.


We owe to Riecke (Wied. Ann. lxvi. 1898) and to Drude (ibid. lxx. 1900) theories of electric and thermal conduction in metals wherein the positive and negative electrons act like the molecules of a perfect gas. Their theories are practically an application of the kinetic theory of gases to free electrons. These theories lead to some valuable results in the physics of electrons, as, for instance, when Drude finds that at a given temperature the kinetic energy of a free electron is identical with that of a molecule of a perfect gas at the same temperature. J. J. Thomson has sketched a similar theory of metallic conduction ('Nature,' May 1900; Congrès International de Physique, Paris, 1900, vol. iii.). These theories give the Wiedemann-Franz law of the approximate proportionality at ordinary temperatures between the electric and thermal conductivities of metals and the variation of their ratios inversely as the absolute temperature. In other words, the kinetic theory of electrons as gas makes the ratio of electric and thermal resistance at any temperature the same for all metals and directly proportional to the absolute temperature. But while the theory fares well with the ratio of the two conductivities, it makes no headway with an account of either conductivity taken separately. The reason is that the unfree electron pairs in the metallic atoms play a prominent part in the mechanism of conduction, which will now be investigated.

We have been led to assume in the last section that the pair of electrons in an atom of metal possesses electrokinetic energy by virtue of the rotation of the electrons round an electric axis; it must therefore have gyrostatic properties. Each atom is equipped with an electrostatic doublet and an electrical gyrostat, both having the same axis. In a metallic conductor all the gyrostats vibrate in a field of electric force
like a number of spinning tops in the field of gravity. During the vibrations it will sometimes happen that the $\uparrow$ and the $\uparrow$ of two adjacent doublets come so near one another that their attractions break up the doublets to form a new one, and liberate $\downarrow$ and $\downarrow$ at about two molecular diameters apart. These in rushing together may be regarded as free, and may upset the stability of other doublets. Thus we must allow to each electron the average free time and free path postulated by Riecke. This dissociation is characteristic of the doublets in metals and causes metallic conduction. The reason for the profound distinction between metals and insulators is that in the monatomic molecule of the metal both $\uparrow$ and $\downarrow$, are in the same atom, whereas in compounds $\uparrow$ and $\downarrow$ are generally in different atoms. For instance, in NaCl we have $\uparrow$ attached to Na and $\downarrow$ to Cl, as proved in the electrolysis of fused NaCl. Thus the $\uparrow$ in Na, if it approaches the $\downarrow$ of the Cl in another molecule, will not break away, because of the decided affinity of the $\uparrow$ for Na and of the $\downarrow$ for Cl. Unless the molecule of NaCl breaks up and allows electrolytic conduction to take place, NaCl acts as an insulator. The monatomic atom of a metal with its doublet is what I have proposed to call a stion. Metallic conduction is due then to the dissociation of stions, electrolytic conduction to the dissociation of molecules. In the case of the Helium family the stions may not dissociate, in which case these elements in the solid state would not conduct like metals. The electrons in the atoms of these elements may be inside instead of on the surface, as appears to be the case with the metals. In insulators, then, each atom confines its own electron or electrons to its own immediate neighbourhood and allows no free time.

In metallic conduction, then, we have to do with the breaking of molecular electrical gyrostats and the flying together of the broken pieces. For simplicity, we can replace the actual discontinuous process of nature by an imaginary equivalent in which each gyrostat preserves a continuous existence, while also a certain number of free electrons travel between the gyrostats, propagating momentum to the same amount as the shocks occurring in the natural process. In a metal of varying temperature the transmission of energy is greater from hot to cold than in any other direction because the free electron carries with it the kinetic energy of the temperature ruling where it gained its freedom, and so we have the conduction of heat. In metals, of course, there will be also the usual molecular conduction of heat as in insulators. On the present occasion we shall treat the conduction of heat
by purely molecular agency in metals as negligible in comparison with that due to electrons.

In the conduction of electricity the gyrostats most affected by an E.M.F. are those having their axes at right angles to the E.M.F., for the couple acting on the electrostatic doublets is greatest when their axes are perpendicular to the E.M.F. Accordingly we shall take as our typical case that represented in the diagram, where the circle in perspective may be taken to be a material flywheel having the same moment of inertia round its axis as the electric gyrostat. It is supposed to be rigidly connected with the doublet axis round which it revolves, the distance between \( a \) and \( b \) being \( s \) and their charges \( e \). The couple due to the E.M.F. acting on the charges is denoted by the arrows. The electrical gyrostat is similar, then, to the familiar experimental case of a flywheel revolving round a horizontal axis supported by a universal joint at one end. The couple due to gravity and supporting force at the joint corresponds to the couple on the electric doublet. The general motion of such a gyrostat is a rotation of the axis in a horizontal plane with small vertical oscillations of the axis. The period of this vibration is \( 2\pi A/C\omega \), where \( \omega \) is the velocity of rotation, and the amplitude of the angular vertical oscillation of the axis is \( 2HA/C^2\omega^2 \), where \( A \) is the moment of inertia of the flywheel round a horizontal axis through the centre of mass and perpendicular to the axis of rotation, and \( C \) is the moment of inertia round the axis of rotation, while \( H \) is the moment of the couple. Since for our pair of electrons \( 2A = C \), we prescribe the same condition for our flywheel, and find its period of vibration to be \( \pi/\omega \) and its amplitude \( H/C\omega^2 \). In the case of our electric gyrostat, if \( X \) is the electric force near it, the couple \( esX \) takes the place of the couple of gravitation for the flywheel. Thus the angular amplitude of the oscillations of the axis of the gyrostat is \( esX/C\omega^2 \). Denote the period of vibration by \( \tau \) which we have seen to be half the period of rotation of the gyrostat, then the average linear velocity of the electrons \( a \) and \( b \) at the end of the axis is \( esX/2C\omega^2\tau \), assuming the centre of inertia to be at rest.

We have found, then, that the effect of an electric force \( X \) on our typical doublet is to give the electrons a to-and-fro motion in the direction of \( X \). We have now to show how these reciprocating velocities produce a continuous velocity
of the \( \sigma \) electrons with \( X \) and of the \( \pi \) electrons against \( X \), so as to give a current proportional to \( X \). The cases in which the \( \sigma \) electron of one doublet is brought near to the like electron of another by the oscillations, or the \( \pi \) electron is brought near to its like, need no further consideration as there is repulsion. But when the \( \sigma \) of one doublet approaches the \( \pi \) of another, the cases in which break up of the doublets occurs must most frequently happen when \( \sigma \) is moving forward with \( X \) and \( \pi \) backward. While the near \( \sigma \) and \( \pi \) of two doublets are forming a new doublet, the remote \( \sigma \) and \( \pi \) of these two doublets are released as free electrons at the end of their vibrations when they are at rest. Therefore they begin at once to move in the field of \( X \). Now, in the theory of Riecke and Drude and Thomson, these free electrons are supposed to travel a certain free path under the force \( X \) and thus maintain the current. This is a possible mode of genesis of the current, but it seems to me more probable that, as these free electrons will rush together to form new doublets, the passing on of \( \sigma \) forward and of \( \pi \) backward at a favourable conjunction of circumstances is the real source of current. As the other action is dependent on this one, its effects may be merged in those of its cause. The result of the oscillatory process, then, is to send forward a certain fraction of the \( \sigma \) electrons and backward the same fraction of the \( \pi \) electrons. Denote this fraction by \( \phi \) and the number of each sort of electrons per unit volume by \( Q \), then we have \( \phi Q \) electrons per unit volume moving forward with velocity \( v \), which is the mean velocity of vibration determined above, the same number moving backwards; so the total current across unit area is

\[
2\phi Qe\tau = \frac{\phi Qe^2s^2X}{\mathcal{C} \omega^2 \tau}. \quad \ldots \ldots (10)
\]

So for the electric conductivity \( \gamma \) we have

\[
\gamma = \frac{\phi Qe^2s^2}{\mathcal{C} \omega^2 \tau}. \quad \ldots \ldots (11)
\]

But \( \mathcal{C} \omega^2/2 \) is the electrokinetic energy of our metallic situation or atom plus doublet. The electrokinetic energy per unit volume is by section 3,

\[
N\theta^2/T\mathcal{T} = (\theta/\tau)\mathcal{I} e^2s^2/\mathcal{K}(2a)^3\}
\]

But \( Q = 1/(2a)^3 \), so

\[
\frac{\mathcal{C}\omega^2}{2} = (\theta/\tau)\mathcal{I} e^2s^2/\mathcal{K}(2a)^3\}, \quad \text{and} \quad \gamma = \mathcal{K} \phi T^2/2 \theta^2 \tau. \quad (12)
\]

We have still to determine \( \phi \) and \( \tau \). We have found that \( \tau \) is half the period of rotation of the doublet. We have pictured the rotation as movement on a curve like a screw
of small pitch on the surface of the atom, so that for finding \( \tau \) it may be treated as though the path of each electron were a circle of radius \( a \). The electric attraction is the centripetal force, and so \( Ie^2/a = e^2/K(2a)^2 \), where \( I \) is the inertia of an electron, assumed to be the same for both electrons; thus

\[
\tau = \frac{\pi a}{v} = 2\pi a^3 I^1 K^1 e. \quad \ldots \ldots (13)
\]

Here \( K \) denotes the dielectric capacity of the stuff of the atom, not quite the same as the average for the atoms and their interspace. It is well worth noticing as another example of duality, that the period thus calculated is proportional to the period of vibration of any one doublet in the field of force due to all the others, on the supposition that the doublet moves free of constraint from the atom, just as we assumed in the above calculation of \( \tau \). The time of vibration being

\[
2\pi \left( \text{moment of inertia} / \text{moment of couple} \right)^{1/2}
\]

is equal to

\[
2\pi \left( \frac{2a^2 I}{es \cdot 4\pi \sigma / K} \right)^{1/2} = 2\pi (\delta a^3 I K / e^2 \pi)^{1/2}
\]

if \( s = 2a, \sigma = e / 4a^2 \), and factor \( 1/3 \) is used.

This is nearly the same as (13). The value of \( \tau \) calculated in each of these ways is independent of the temperature, and therefore of the electrokinetic energy impressed on the doublet by the atom. It is the same as the period calculated on the assumption that all the electric energy of a doublet is kinetic energy; for if \( \Omega \) is the velocity of rotation for this, then

\[
\frac{1}{2} C \Omega^2 = N = 2\pi e^2 \delta / 3K(2a)^2, \quad \text{and} \quad C = 2a^2 I,
\]

\[
\therefore \quad \tau = \frac{2\pi}{\Omega} = 2\pi (\delta a^3 I K / e^2 \pi)^{1/2}.
\]

Substituting in (12) for \( \tau \) the value given in (13), we have

\[
\gamma = K^1 \phi T e^2 / 4\pi \theta a^3 I^1. \quad \ldots \ldots (14)
\]

To express \( \phi \) the simplest assumption we can make is that the chance of the occurrence of conditions favourable to the breaking up of two adjacent doublets will be proportional to the ratio of the free spaces between the atoms to the volume of the atoms, this ratio being a measure of the freedom of the atoms to move. In section 3 we took the angular velocity of the atoms to be proportional to \( b\theta \). Thus
with \( b \) as coefficient of linear expansion we take \( \phi \) proportional to \( b^2 \), equal to say \( Gb^2 \), and then

\[
\gamma = K^2 Gb^2 T^2 e / 4 \pi \theta a^2 b. \quad \ldots (15)
\]

This expression for the conductivity has been obtained by the consideration of an atom which contains only one doublet. Now there is reason for believing that the monatomic molecule of a divalent metal such as calcium or zinc contains two doublets, and generally that the number of doublets in the molecule of a metal is equal to its valency (Phil. Mag. [5] xxxix.). So far, then, we have considered only the case of a monad metal, for we put \( Q = 1 / (2a)^2 \). For a metal of valency \( v \) we cannot treat the \( v \) doublets like entirely separate entities, as though they were in separate atoms, nor can we simply change the \( e \) of the previous calculations into \( ve \). We must remember that all the doublets in a polyvalent metal atom have a resultant electric moment, and that if we still denote this by \( \{es\} \), \( e \) and \( s \) separately have not their former meanings. Thus in the process of calculating \( \gamma \) we used \( esX \) for the couple acting on a doublet. But the \( v \) doublets in an atom having a moment \( \{es\}X \), we ought for the whole electrical system of the \( v \)-valent atom to use \( \{es\} \) instead of the simple \( es \). But in "Further Studies" there are two lines of evidence that \( \{es\}^2 \) has only \( v \) times the value of \( es^2 \). Since we are now treating \( s \) as equal to the molecular diameter, it follows that if in \( \{es\} \) we still treat it so, then \( e \) in \( \{es\} \) has only \( v^2 \) times the value of \( e \). But at each vibration of the electrical system of an atom, it is most likely that only one electron takes part in the transmission of current, and not \( v^2 \) electrons. \( Q \) is still equal to the number of atoms per unit volume, and \( \phi \) remains the same. Consequently for \( v \) doublets in an atom \( es^2 \) in (10) should be replaced by \( \sqrt{v} es^2 \), while the moment of inertia for the \( v \) doublets becomes \( v \) times that for a single one. Hence, when \( es^2 \) is eliminated as in (12) we shall get

\[
\gamma = K\phi T^2 / 2\theta e v. \quad \ldots \ldots (16)
\]

The period of vibration \( \tau \) is not affected, because couple and moment of inertia are increased \( v \) times. So for a metal of valency \( v \) (15) becomes

\[
\gamma = K^2 Gb^2 T^2 e / 4 \pi \theta a^2 b v^2. \quad \ldots (17)
\]

Now, in "Further Studies" it was shown in Table xxix. that \( M^2 \) for the main families is proportional to \( v(M/\rho) \) and in the subordinate families to a simple multiple of \( v(M/\rho) \).
But \( M^2 \) is proportional to \( e^2 s^2 / K \), and in the metals we have found \( s = (m / \rho)^{1/3} \). If, then, we take \( K \nu (m / \rho)^{1/3} \), that is \( K \nu a \), to be a constant \( \beta^2 \), we eliminate \( K \) from (17) thus

\[
\gamma = G \beta \nu T^2 / 4 \pi \theta a^2 v I^4 \quad \ldots \ldots \quad (18)
\]

As \( I \) the inertia of an electron has been assumed constant, and \( 1 / \gamma \) is the electrical specific resistance at \( \theta \), this can be written

\[
\frac{1}{\gamma} \left( \frac{M}{\rho} \right)^{1/3} b T^2 \left( \frac{M}{\rho} \right) \nu = \text{constant} \quad \ldots \ldots \quad (18a)
\]

This is the relation discovered by W. Williams (Phil. Mag. [6] iii. 1902). It must be noted that several metals like Ag, Cu, Tl, and Pb, which have \( \beta^2 \) replaced by a simple multiple of \( \beta^2 \), show no appearance of the effect of the simple multiplier in the formula of Williams, while some metals are exceptions to this rule. It is possible that for these two classes of exceptional metals \( I \) cannot be assumed to be constant, but that the ratio of the simple multiplier of \( \beta^2 \) to \( I \) is constant or a simple number. On account of the importance of (18a) I shall reproduce the data gathered by Williams with the addition of a few, to show how nearly the left side of (18a) is the same for most of the metals. That equation makes resistance proportional to absolute temperature, which is the well-known approximate truth pointed out by Clausius. Our comparison will be restricted to the case where \( \theta = 273 \), unless it is otherwise stated. The resistance \( 1 / \gamma \) is given in ohms for a cm.\(^3\) For \( \nu \) the highest known valency is given.

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<td></td>
</tr>
<tr>
<td>10M/\rho ...</td>
<td>237</td>
<td>454</td>
<td>71</td>
<td>102</td>
<td>101</td>
<td>139</td>
<td>254</td>
</tr>
<tr>
<td>10^6 b ...</td>
<td>720</td>
<td>30</td>
<td>170</td>
<td>194</td>
<td>147</td>
<td>270</td>
<td>(279)*</td>
</tr>
<tr>
<td>T ........</td>
<td>369</td>
<td>335</td>
<td>1333</td>
<td>1173</td>
<td>1310</td>
<td>1023</td>
<td>853?</td>
</tr>
<tr>
<td>( \nu ) ........</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>22</td>
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<td>25</td>
<td>31</td>
<td>13</td>
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<tbody>
<tr>
<td>58</td>
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<td>210</td>
<td>29</td>
<td>84</td>
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<td>141</td>
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<td>153</td>
<td>172</td>
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</tr>
<tr>
<td>208</td>
<td>316</td>
<td>(778)*</td>
<td>222</td>
<td>417</td>
<td>302</td>
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</tr>
<tr>
<td>676</td>
<td>593</td>
<td>234</td>
<td>923</td>
<td>449</td>
<td>561</td>
<td>503</td>
</tr>
<tr>
<td>( \nu ) ........</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
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<td>28</td>
<td>34</td>
<td>14</td>
<td>14</td>
<td>31</td>
</tr>
</tbody>
</table>

* Calculated by empirical formula \( bTM^1 = 044 \).
Mr. W. Sutherland on the

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>107/γ</td>
<td>202</td>
<td>351</td>
<td>431</td>
<td>1080</td>
<td>102</td>
<td>90</td>
<td>110</td>
</tr>
<tr>
<td>10M/ρ</td>
<td>181</td>
<td>132</td>
<td>179</td>
<td>211</td>
<td>92</td>
<td>91</td>
<td>72</td>
</tr>
<tr>
<td>10b</td>
<td>390</td>
<td>(278)*</td>
<td>116</td>
<td>137</td>
<td>110</td>
<td>91</td>
<td>118</td>
</tr>
<tr>
<td>T</td>
<td>695</td>
<td>775</td>
<td>710</td>
<td>538</td>
<td>1775</td>
<td>2050</td>
<td>2350</td>
</tr>
<tr>
<td>υ</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

* Calculated by empirical formula bTMe^2 = 0.44.

The last row in this table contains the values of (10^3/γ)T^2/θ(M/ρ)υ, and at a first glance they do not seem to be near enough to a constant to verify (18 a), but the row contains some metals having such exceptional characters as demand modification of the theory leading to (18 a). First, As is a metal with properties merging into those of a non-metal. Its molecule in the vaporous state is As4. On this account As ought to be excluded from a table of normal metals. Although Sb and Bi, two other members of the As family of metals, would seem to fall into line as normal metals, I am dubious about 5 being the right value of υ to use. Fe and Ni ought to be excluded from the above table on account of their magnetic properties, which indicate a special action and reaction between the revolving doublets and the atom. The magnetic metals require a special theory. The other metals giving distinctly exceptional values are Au, Al, In, and Sn. In their case the discrepancy I believe to be due to the fact that in these atoms the number of doublets is not identical with the valency of the atom. It is noteworthy that three out of the four trivalent metals should contribute values 13, 14, 14 to the row. The exceptional results for Au, Al, and In can be accounted for by a principle which I have discussed in connexion with the tetrad valency of oxygen and the constitution of water in "Ionization, &c." For example, the pentad valency of N was traced to its atom having associated with it four ḫ electrons and one ḫ. If the ḫ unites with one ḫ to form a doublet the N atom can act as a triad, as in ammonia. If, then, the pentad metals Sb and Bi behave similarly, we might expect their atoms in the element state to form three doublets corresponding to the triad valency in addition to the one doublet formed by the union of ḫ and ḫ. Perhaps, then, for Bi and Sb in the above table, 4 should be substituted for 5 in the values of υ. Returning then to the triad atoms of Au, Al, and In, we could ascribe their valency to two ḫ and one ḫ electron. For auric chloride AuCl₃, the graphic formula would be as shown. Its reduction to aurous chloride AuCl is readily explicable by
Electric Origin of Rigidity and Consequences.

This formula, since Cl_{2} and Cl_{2} supply everything to make Cl_{2}, and the $\#$ of Au unites with a $\#$ to form a doublet, and leaves Au to act as monovalent with one free Cl. It is possible, then, that in metallic gold the atom has two doublets. Similarly with Al and In. These three triad metals fall into line with the others if their atoms contain one or two doublets instead of the three assigned to them in the table. In the case of Sn the tetravalency of the atom may be due to two $\#$ and two $\#$ electrons. This would give the atom of metallic tin two doublets and would bring Sn into a less exceptional position. One fact is very suggestive of truth in our speculations about the valencies of Au, Al, and Sn, namely, that it is so difficult to determine their ionic velocities that even yet values are not given. The interesting chemical inquiries suggested cannot be followed up here. We have shown good cause why the four smallest values in the last row of our table should not be held to be at variance with the principles at the basis of (18 a), but should lead to a means of estimating the actual number of doublets in an atom of uncombined metal more refined than that of treating it as the same as the highest valency in all cases. These considerations throw doubt on the value for Tl and Pb. The case of Cu is also not quite clear. But in considering the table as a whole, it must be remembered that $10^{7}/\gamma$ ranges from 15 for Ag to 1080 for Bi, that $\nu$ ranges from 1 to 8, $M/\rho$ from 6.7 to 45.4, $10^{7}b$ from 91 to 830, and $T$ from 234 to 2350, $T$ entering also as a square. The approach to constancy in these circumstances is such as to show a substantial soundness in the train of thought leading to (18 a).

In the metallic conduction of heat, we shall at present ignore that part of it which is identical with the molecular process of heat-conduction, as it takes place in electric insulators, and investigate only the chief part effected by the electric doublets. In electric conduction we regarded the electric gyrostats as forced to vibrate in a field of electric force $X$. But in a mass of metal free from electric force and at a uniform temperature each doublet is thrown into oscillations of a certain mean frequency because of atomic motion. These produce conjunctions favourable for the breaking of doublets and the forming of new ones. This process causes a movement of electrons through the mass of metal which is very similar to that of the molecules in a perfect gas. If there is variation of temperature in a metal, the motion of the electrons is the cause of a flow of heat from the hot parts to the cold. Riecke,
Drude, and Thomson treat the conduction of heat by electrons as practically identical with that by molecules of a perfect gas. In the present theory, although the breaking up and renewal of doublets causes a free movement of electrons in all directions, these movements go on not according to the laws of molecules in a gas, but according to the laws of electric gyrostats. Just as electric conduction is due to directed vibrations of the gyrostats in the direction of electric force, thermal conduction is due to thermal vibrations of the gyrostats maintained equally in all directions. The mean effect of a collision between two atoms is to leave each with the same kinetic energy but differently directed. The axis of rotation of each atom is also changed in direction. Thus, there must be set up couples of action and reaction between atom and electric gyrostat, just as if the gyrostat were a material one revolving round an axis with bearings in the atom. As the kinetic energy of rotation of the atom remains on the average constant and only changes the direction of its axis, the moment of each couple must on the average be proportional to the kinetic energy of rotation, and therefore to the kinetic energy of translation of the atom. Thus in place of the electric couple $esX$ we have a thermal couple which we may write $Amv^2$, $A$ being a constant. In place of the angular amplitude $esX/C\omega^2$ of the electric vibration of the gyrostat we have $Amv^2/C\omega^2$ for the angular amplitude of the thermal vibration of the gyrostat. For the mean velocity of each electron due to the thermal vibration we have

$$Amv^2/2C\omega^2\tau. \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (19)$$

Now each electron which is passed on from one molecule to another advances a distance $2a$. If there is a fall of temperature in any direction, one-third of the movements of electrons may be assumed to take place only in that direction, the rest perpendicularly to it. An electron moves from a place where it is in thermal equilibrium with an atom having kinetic energy $mv^2/2$ to another place where it comes into thermal equilibrium with an atom having energy $mv^2/2 + 2ad(mv^2/2)/dx$, $x$ being in the direction of motion. The electron carries forward such fraction of the energy $2ad(mv^2/2)/dx$ as its own energy is of $mv^2/2$. Call this fraction $f$. Then the current of heat across unit area perpendicular to $x$ is

$$\frac{2}{3}\phi Q a \Lambda (mv^2/s/C\omega^2\tau)f \frac{d(mv^2/2)}{dx} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (20)$$

Now $mv^2/2 = \alpha \theta$, where $\alpha$ is generally taken to be constant, but we shall see that for the metals it is not quite so.
Electric Origin of Rigidity and Consequences.

For the thermal conductivity \( k \) we have with \( s = 2a \),

\[
 k = \frac{3}{2} \phi Q(2a)^2 f A \theta^2 / C \omega^2 \tau, \ldots \ldots \quad (21)
\]

\[
 \therefore \text{from (11)} \quad k/\gamma = 4f A \theta^2 / 3 e^2, \ldots \ldots \quad (22)
\]

This is the central relation of the theory of Riecke and Drude. It embodies the Wiedemann-Franz law. Drude's theory makes \( f \) identical with \( \alpha \) and \( \lambda = 1 \). Then, with the values of \( \gamma \) and \( k \) for silver at 18° C. and J. J. Thomson's value of \( e \) as \( 6 \times 10^{-10} \), he finds for \( \alpha \) the value \( 26.5 \times 10^{-17} \), while in the kinetic theory of gases it is \( 5.6 \times 10^{-17} \). As Thomson has found \( 3 \times 10^{-10} \) to be a better estimation of \( e \), Drude's value of \( \alpha \) becomes \( 13 \times 10^{-17} \). This is near enough to the kinetic theory value to show that Drude's assumption that the kinetic energy of an electron equals that of a molecule is substantially correct. Thus the free electron of the Riecke theory at a given temperature has the same kinetic energy as a molecule of gas at that temperature. The present theory does not necessitate the literal truth of Drude's principle. All that we need to assume is that an electron in moving from one place to another carries to its molecule in the second place an amount of energy nearly equal to its excess or defect over the molecular energy at the first place. The simplest way of giving the electron this property is to treat it as a molecule of a perfect gas, but it is not the only way. By means of the data of Jaeger and Diesselhorst (Sitz. d. k. pr. Akad. der Wiss. Berlin, 1899) for \( \gamma \) and \( k \) for a number of pure metals on the same specimens at 18° and 100°, Drude shows that while according to (22) the ratio of \( k/\gamma \) at 18° and at 100° is \( 373/291 = 1.28 \), the experimental values range from 1.26 for Sn to 1.35 for Pt and Pd, the mean for 13 metals being 1.30. For the exceptional metal Bi it is 1.12. But according to (22) \( k/\gamma \) at a given temperature ought to be the same for all metals. Here are the values given by Jaeger and Diesselhorst:

<table>
<thead>
<tr>
<th>Metal</th>
<th>( k/\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>636</td>
</tr>
<tr>
<td>Cu</td>
<td>605</td>
</tr>
<tr>
<td>Ag</td>
<td>686</td>
</tr>
<tr>
<td>Au</td>
<td>727</td>
</tr>
<tr>
<td>Ni</td>
<td>699</td>
</tr>
<tr>
<td>Zn</td>
<td>706</td>
</tr>
<tr>
<td>Cd</td>
<td>715</td>
</tr>
<tr>
<td>Pb</td>
<td>735</td>
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<td>Sn</td>
<td>753</td>
</tr>
<tr>
<td>Pt</td>
<td>754</td>
</tr>
<tr>
<td>Pd</td>
<td>802</td>
</tr>
<tr>
<td>Fe</td>
<td>862</td>
</tr>
<tr>
<td>Bi</td>
<td>962</td>
</tr>
</tbody>
</table>

If we except Fe for its strongly magnetic properties and Bi on account of its crystalline structure, we find \( k/\gamma \) increase from 636 for Al to 754 for Pd. It is of interest that for these same metals the atomic heat \( M \) increases in about the same proportion, as the following values of \( 10M/c \) show when arranged for the metals in the same order as above. The values of \( c \) used are Regnault's at about 60° C.

<table>
<thead>
<tr>
<th>Metal</th>
<th>( 10M/c )</th>
</tr>
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<tbody>
<tr>
<td>Al</td>
<td>605</td>
</tr>
<tr>
<td>Cu</td>
<td>686</td>
</tr>
<tr>
<td>Ag</td>
<td>727</td>
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<tr>
<td>Au</td>
<td>699</td>
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<tr>
<td>Ni</td>
<td>706</td>
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<tr>
<td>Zn</td>
<td>715</td>
</tr>
<tr>
<td>Cd</td>
<td>735</td>
</tr>
<tr>
<td>Pb</td>
<td>753</td>
</tr>
<tr>
<td>Sn</td>
<td>754</td>
</tr>
<tr>
<td>Pt</td>
<td>802</td>
</tr>
<tr>
<td>Pd</td>
<td>862</td>
</tr>
<tr>
<td>Fe</td>
<td>962</td>
</tr>
</tbody>
</table>

Mr. W. Sutherland on the

ratios \( \frac{k}{\gamma M} \) is added to show the proportionality under
discussion.

<table>
<thead>
<tr>
<th>Element</th>
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<th>14</th>
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<th>20</th>
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<tbody>
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<td>Al</td>
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<td>60</td>
<td>61</td>
<td>64</td>
<td>64</td>
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<td>63</td>
<td>64</td>
</tr>
<tr>
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<td>65</td>
<td>62</td>
<td>63</td>
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<td>65</td>
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<td>64</td>
</tr>
<tr>
<td>Au</td>
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<td>109</td>
<td>110</td>
<td>111</td>
</tr>
<tr>
<td>Cd</td>
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<td>120</td>
<td>125</td>
<td>120</td>
<td>120</td>
<td>125</td>
<td>148</td>
</tr>
</tbody>
</table>

By taking account of the molecular heat we get a decidedly
more accurate relation between thermal and electrical con-
ductivity than is given by the Wiedemann-Franz law. Now
this improvement on that law is in reality contained in the
theory just given of thermal conduction, for we assumed
 provisionally for simplicity that \( a \) is constant in the equation
\( \frac{mv^2}{2} = a \theta \). But for the metals we should take \( a \) to be pro-
portional to \( M_e \). Then (22) gives at once that \( \frac{k}{\gamma M} \) is to
be the same for all metals, as we have just found it to be to a
considerable degree of approximation. Of course the atomic
heat at 18° and 100° ought rightly to enter into the com-
parison of \( k/\gamma \) at these temperatures, and would perhaps
improve the results of Drude's comparison.

Riecke and Drude make some enterprising and elaborate
attempts to build up a theory of thermoelectricity on the
perfect gas analogy for free electrons, treating of the Peltier,
Thomson, Hall, and Nernst and Ettingshausen effects. I do
not propose at present to apply the theory of electric gyro-
stats to these subjects, because Liebenow (Zur Thermodynamik
der Thermoketten, Wied. Ann. lxviii. 1899) has sketched a
most promising thermodynamic deduction of the Peltier and
Thomson effects from the electrical and thermal conductivities
of metals. So a successful molecular theory of thermoelec-
tricity presupposes a satisfactory electronic theory of the
Second Law of Thermodynamics as applied to electricity.

Moreau has shown (Compt. Rend. cxxx. 1900) that the Nernst
and Ettingshausen effect can be deduced from the Thomson
and Hall effects. There is reason to believe that the Hall
effect is connected with the electric and magnetic relations
of elasticity, and as we have shown that rigidity in its origin
is electrostatic and its temperature variation is electrokinetic,
it is evident that a satisfactory electronic account of the Hall
effect probably involves a complete theory of the effects of
electric and magnetic fields of external origin on the elastic
properties of metals. This large subject, first opened up by
Kelvin half a century ago, has probably to be worked up
before we get a satisfactory theory of the Hall effect. It is
interesting to reflect that Kelvin, in applying the laws of
thermodynamics to electricity, many years ago foreshadowed
the fundamental similarity between matter and electricity.
which is now the source of so much fruitful research in physics. It seems to me that the magnetic properties of metals might be the most promising field in which to test further the capabilities of the electric gyrostat for explaining the relations of matter and electricity.

We can apply a partial check on the results of this section by using (12) to calculate the order of magnitude of $\phi$ the fraction of the doublets occupied at any time in the transmission of electric current. Take the case of silver, for which at $\theta = 273\,\text{K}$ is given in the Table along with $T$. For the silver ion $K$ is 2.66, and if we take this as a value for metallic Ag, then from (12) we find $\tau$ to be of the order $4 \times 10^{-14}\phi$. But in "The Cause of the Structure of Spectra" I have shown that the fundamental period in the vibrations of atoms is of the order $10^{-14}$, and so $\phi$ is of the order $1/4$. Not much weight is to be attached to this evaluation, but if $\phi$ had come out a large integer, say $10^6$, there would have been strong reason to doubt the cogency of the preceding calculations.

5. Contact Difference of Potential, and Mechanism of the Storage of Potential Chemical Energy in the $\text{Æ}$ther.

The question of the contact difference of potential in metals is one that Lodge has kept to the fore during the last twenty years. It is fundamental in electricity and chemistry. The special point emphasized by Lodge is that the contact E.M.F. of metals can be calculated from their heats of combustion in oxygen in the following way. Conceive a gramme equivalent $M_1$ and $M_2$ of each of two metals to be burnt in oxygen, giving heat $h_1$ and $h_2$. Let $q_1$ and $q_2$ be their electrochemical equivalents, then the quantity of electricity taking part in their burning is $M_j/q_j$. If, then, $V_1 - V_2$ is the contact difference of potential for these two metals, the electrical work done by the quantity $M_1/q_1$ in this change of potential would be $(V_1 - V_2)M_1/q_1 = V_1M_1/q_1 - V_2M_2/q_2$. On equating this to $h_1 - h_2$ we get Lodge's principle,

\[(V_1 - V_2)M_1/q_1 = h_1 - h_2.\quad \ldots \quad (23)\]

Lodge considers that the oxygen atoms in straining at the metal atoms establish the contact E.M.F. If the oxygen could be completely got rid of, and the metals be immersed in perfectly dry chlorine, their contact E.M.F. ought according to Lodge's idea to be calculated from the heats of combination of the metals with chlorine. The evidence as a whole seems to me to be against this particular view, but the important matter is that the contact E.M.F. for a pair of
metals is of the same order of magnitude and in about the same succession for different pairs of metals, as if calculated from the heats of combination with oxygen, chlorine, or some other powerfully electronegative element. Helmholtz's electron theory of chemical valency furnishes a simple explanation of this principle, if it is suitably modified by making the doublet rather than the electron the main agent. From "The Electric Origin of Molecular Attraction" (Phil. Mag. [6] iv.), and previous papers on molecular force, it follows that the doublet of a compound inorganic molecule such as NaCl consists of two parts, one belonging to Na and the other to Cl. This means that the doublet of NaCl is made up of two doublets, just as a single bar-magnet can be made out of two. Reinganum (Ann. d. Ph. [4] x.) has proposed the use of doublet in place of electron. Helmholtz's original view of chemical valency can be expressed by writing the formula for NaCl in the form Na$^\pm$Cl. But when we assign to Na and Cl a single doublet each, in place of the single electron, the formula becomes $^\pm$Na$^\pm$Cl$^\pm$. The inner pair of electrons are so close to one another that their external effects are neutralized, and the total external effect is that due to the two outer electrons which act as a single doublet.

The diagram makes clear the importance of the substitution of doublets for electrons as the agents in chemical combination. The points A, B, C, D denote the positions of the four electrons whose symbols are placed over them.

The distance BC being negligible in comparison with AB and CD, the electric moment of the whole molecule is $e \cdot AD$, to which Na contributes the moment of its own doublet $e \cdot AB$, and Cl the moment $e \cdot CD$. The investigation of the values of these parts has occupied a considerable portion of my papers on molecular attraction. Now the very smallness of BC makes it of fundamental importance in the electrical theory of thermochemistry, for the electrons $\mathbf{\#}$ at B and $\mathbf{\#}$ at C have their electric energy expressible as $e^2/BC^2$. Thus BC, which is neglected in the study of molecular attraction, becomes the principal physical quantity in the thermochemistry of atoms. Potential chemical energy is stored by increasing the potential energy of inner doublets like BC.

The equation for the reaction between Na and Cl must now be written

$$2\#Na + \#Cl \rightarrow \#Na\#Cl \quad \ldots \quad (24)$$
The form given to Cl₂ in this equation shows that the dissociation of Cl₂ into 2Cl, or of I₂ into 2I does not involve the production of ions, but only of stions. Thus the vapour of I₂ containing free atoms I is not necessarily a conductor, nor is that of N₂O₄, and therefore the difficulty about Richarz's application (Wied. Ann. liii.) of Helmholtz's theory to the dissociation of N₂O₄ and of I₂ disappears, if we replace the electron of Helmholtz by the doublet. The doublet theory greatly simplifies the constitution of all the diatomic element molecules such as H₂ and N₂, for it makes both atoms of the molecule similarly equipped, instead of making one positive and the other negative as the electron theory does. On the other hand, it is necessary to assign some reason why no more than two doublets should unite to form these molecules, and why the metal monatomic molecules retain their individualities. However, the chief difference that the doublet theory of valency makes when substituted for the electron theory is, that for every doublet which the electron theory would require on account of the attraction of opposite electrons, the doublet introduces an inner doublet whose electrons are very close together. Thus the graphical formulæ proposed for water, NH₄Cl, and PCl₅, in "Ionization &c.," are easily adapted to the doublet theory, which raises very nice problems of stable configurations in space of three dimensions analogous to those studied experimentally by Mayer and Wood (Phil. Mag. [5] xlvi.) for small magnets in a plane. In the electrical theory the molecule becomes similar to a multipolar dynamo armature.

The doublet theory gives two possible isomeric forms for NaCl, namely, §Na##Cl§ and ¥Na ¥Cl ¥. We can express the electropositive nature of Na and the electronegative of Cl by stating that the latter of these forms is the only one that occurs. But the fact that an atom may be electropositive to a second atom and electronegative to a third can be simply expressed by formulæ like those just written. The ionization of NaCl may be described as the removal of the internal doublet from §Na§Cl§ by means of the solvent, and the separation of the two parts §Na and Cl§ to such a distance that they cannot immediately recombine. We may regard an ion like §Na as a stion §Na § with an added § which coincides with the § and neutralizes it completely. The advantage of this method of regarding an ion is that it assigns to the ion the doublet which gives the power of cohesion, and the ions in solution behave as though they take part in the cohesion of the solution. According to this view of the ion, it would be better to describe the ionization of
Mr. W. Sutherland on the

NaCl as the introduction of a doublet in the position shown thus $\text{NaCl} \equiv \text{Na}^+\text{Cl}^-$, the two inner doublets reacting parallel to the thick line, while neutralizing one another's chemical cohering power, and so allowing the molecule to split into $\text{Na}^+$ and $\text{Cl}^-$, which are the two ions. The central problem of ionization is to discover whence the extra doublet is derived. We may of course write the formula for NaCl in the form $\text{Na}^+\text{Cl}^-$, and so equip the molecule with three doublets, which during ionization are resolved into two separate doublets, each of which has associated with it one of the electrons into which the third doublet has been broken.

To follow out this line of thought and give a logical representation of the monatomic molecule of metallic Na, we should write its symbol thus $\text{Na}^+$, indicating that there are two doublets present, one having an electric moment which is negligible in comparison with that of the other. Similarly, the symbol for Cl will become $\text{Cl}^-$.

It will not be necessary to use such long formulae continually, if we remember that $\equiv$ may stand for any number of associated doublets, the preferable number being the smallest that will enable all the known facts to be logically set forth.

With this notation we can now discuss the separate parts of the energy changes whose total forms the heat of the chemical reaction given in (24). The formation of $2\text{Na}^+\text{Cl}^-$ from Cl$_2$ and 2Na involves the work of splitting $\text{Cl}_2$ into $\text{Cl}^-$, which may be denoted by (S), where S means any electronegative element, that of bringing $\text{Na}^+$ and $\text{Cl}^-$ into the position $\text{Na}^+\text{Cl}^-$, involving work depending on the distance between $\equiv$ and $\equiv$ in the inner doublet; this work may be denoted by 2(RS), and then in accordance with Helmholtz's idea of the specific attraction between matter and electricity there is the work of bringing the $\equiv$ of Cl quite close up to Na and that of bringing the $\equiv$ of Na quite close up to Cl. Now, in the more powerfully electronegative metals the changes of volume of their atoms on combination with Cl, Br, and I are approximately the same; and this fact suggests that the work of bringing $\equiv$ of Cl quite close up to Na is approximately independent of the electronegative reagent, and may be written (R) with R as general symbol for a metal. The corresponding quantity for Cl may be merged in (S). The work (R) is sometimes accompanied with quite a remarkable change in the volume of the metal atom. For the gramme atom of Li the change on passing from the metallic to the combined state is 11.9-2; for Na it
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is 23·7-7·4, and so on. There is evidence (see "Ionization &c.") that a gramme atom of Li ions retain the volume 2 characteristic of the combined state, of Na ions the volume 7·4 and so on. Hence it appears that the contiguity of $\#$ to Na causes a large reduction in its volume. On the other hand, any change of volume in the Cl and Br atoms on passing from Cl$_2$ and Br$_2$ into compounds is relatively small, and in the case of I the change seems to be an expansion instead of a contraction. It would appear that when $\#$ and $\&$ attract one another in a metallic atom, the atom acquires such a volume as makes centrifugal force equal to the attraction. When the attraction is neutralized the atom collapses to a much smaller volume. The simplest assumption we can make as to the mutual potential energy of a metal atom and $\#$ and $\&$ is that Na and $\#$ have minimum potential energy in contact, Na and $\&$ when apart, that is Na and $\#$ attract, Na and $\&$ repel. In metallic Na the $\&$ would be expelled but for the direct attraction of the $\#$. The simplest assumption we can add as to the relation between the attraction and repulsion is that they are equal, or that the mutual potential energies of Na and $\#$ and of Na and $\&$ are equal in magnitude but of opposite sign. If there is a strong enough external electric field to counteract sufficiently the attraction between $\#$ and $\&$, the repulsion of the metal atom for $\&$ will drive it out, giving the phenomena of the cathode rays. In a similar way the Becquerel rays may originate.

Returning to the energy changes to go with (24) we get for the heat of reaction

$$2(R) + S + 2(RS) = 2h \quad \ldots \ldots \quad (25)$$

For two metals we have

$$(R_1) - (R_2) + (R_1S) - (R_2S) = h_1 - h_2 \quad \ldots \ldots \quad (26)$$

Now, at a junction of these same two metals we may consider the passage of a current to be the exchange across the junction of $\#$ for $\&$. By the loss of $\#$ the first metal gains potential energy $(R_1)$ and by the gain of $\&$ it gains energy $(R_1)$, and so for the passage of $2e$ across the junction the gain of energy is $2(R_1) - 2(R_2)$. Consequently there must be an E.M.F. $V_1 - V_2$ across the junction such that the total energy change of the $2e$ crossing the junction is nil;

$$e(V_1 - V_2) = R_1 - R_2 \quad \ldots \ldots \quad (27)$$

Comparing this with (26) we see that if $(R_1S) - (R_2S)$ is of subordinate importance to $(R_1) - (R_2)$, Lodge's principle will be nearly true, namely, $e(V_1 - V_2) = h_1 - h_2$. Now, in
“The Fundamental Atomic Laws of Thermochemistry” (Phil. Mag. [5] xl.) I have shown that (RS) is of secondary importance to (R). It appears then that Lodge’s principle concerning the Volta Contact E.M.F. is the outcome of certain laws in the mutual energetics of matter and electricity. It should be noticed that although (RS) was originally defined as the work of bringing R and its attached doublet into combination with S and its attached doublet, it is possible that it may contain parts depending on R alone and on S alone, these merging into (R) and (S) and leaving a comparatively small part depending on both R and S. This part, which in the thermochemical paper is denoted by f(RS), is what is of secondary importance, and not the complete (RS). The further investigation of the energetics of matter and electricity would carry us too far into thermochemistry, which must be left for separate treatment. The object of this section has been mainly to show the relation of the Volta Contact E.M.F. to atomic energies.

6. The Neutron Structure of the Æther used for Calculation of its Density and Rigidity, and Deduction of the Velocity of Light.

The electric doublet must be the basis of the electric and magnetic properties of the æther. The doublet of the æther, which I have proposed to call the neutron, may be represented as $\#$ and $\bar{\#}$ in contact in the form of spherical shells of electricity of radius $\alpha$. The electric moment of the neutron is $2ae$. The inertia of electricity, contemplated by Maxwell and first calculated by J. J. Thomson, necessitates the existence of a definite amount of inertia in each neutron, and so necessitates a definite density of the æther. Moreover, if the neutrons are packed closely enough like the molecules of solid bodies, they will confer rigidity on the æther. An æther is conceivable having its neutrons free like the molecules of a gas, but exercising strong mutual directive influences through their polarities. For rapid shears this would act like a solid, for slow displacements like a gas. Thus from the electromagnetic properties of the electron we have deduced density and rigidity as essential properties of the æther. The postulates of the elastic and of the electric theories of light are but different expressions of the same things. Now the magnitude of $\alpha$ was estimated in “Cathode, Lenard, and Röntgen Rays” (Phil. Mag. [5] xlvii.) as $10^{-14}$ cms. on the assumption that the ratio $I/e$ of the inertia $I$ of a cathode projectile to its charge $e$, determined by J. J. Thomson, was
purely the ratio of the inertia of an electron to its charge and did not involve any "mass of a particle." The experiments of Kaufmann (Compt. Rend. Oct. 1902) have since proved that the mass associated with the electron in cathode rays is entirely electromagnetic. Several subsequent estimations have been made of $\alpha$ by various authors on the assumption that the inertia of the electron is purely electric. Kaufmann gives $10^{-13}$ cms., while Abraham in his comprehensive paper (Ann. d. Ph. [4] x. 1903) writes

$$10^{-13} < \alpha < 10^{-12}.$$ (28)

The equation by which these values have been calculated is that of J. J. Thomson (to a constant près)

$$I = 2e^2/3\alpha v^2,$$

where $v$ is the ratio of the two units of electricity.

If this inertia belongs to a cube of ether of edge $2\alpha$ the density $\rho$ of the ether is

$$\rho = e^2/12\alpha v^2.$$ (29)

Now for the ether rigidity will be determined exactly as for a metal at absolute zero. The rigidity $N = \sigma^2$, where $\sigma$ is the surface-density of the laminar distributions of electricity equivalent to the neutrons. So, just as in section 1, we write

$$\sigma^2 = \frac{2\pi}{3} \left( \frac{e}{4\alpha^2} \right)^2, \quad \therefore N = 2\pi e^2/48\alpha.$$ (30)

The density and rigidity of the ether being found, we get the velocity of light by the formula

$$V = (N/\rho)^{1/2} = e(\pi/2)^{1/2} = 1.25c.$$ (31)

With $\rho = 1/4\pi\alpha^3/3$ we would get $V = e\pi/12) = 1.90c.$ (32)

The coefficient of $e$ ought to be 1, but numerical difficulties occur in connexion with the arbitrary adoption of spheres and cubes as standard shapes. The important point is that by exactly the same process as was applied to matter a rigidity has been found for the ether which, along with the density, gives the velocity of light. It should be noticed that $N$ and $\rho$ both vary as $\alpha^{-4}$, and so $\alpha$ disappears in $V$. With $e$ of the order $3 \times 10^{-16}$ and $e/\lambda = 6 \times 10^{17}$, we have $\alpha$ of the order $2 \times 10^{-13}$. Hence from (29) and (30) we get the values

$$\rho = 5 \times 10^6, \quad N = 8 \times 10^{30}.$$

These huge values for the density and rigidity of the ether result from the small value of $\alpha$ being raised to the power $-4$. They may be taken as upper limits, for an ether made of
electrons in contact. If the electrons in the æther, instead of being in contact, formed doublets at the centre of massless spheres made of æther, the density and rigidity of the whole æther due to the doublets would be reduced by a factor obtained by raising the ratio of $\alpha$ to the radius of the æther sphere to the power 4. We have of late, however, been familiarized again with the conception of a very dense and proportionally rigid æther (for example, Reynolds, 'Scientific Papers,' vol. iii., finds for his granular medium a density $10^9$).

At present we are concerned with working out the consequences of the electron theory. On being applied to the æther it leads to the above density and rigidity, which cannot be dismissed for their absurdity merely because of their magnitude. Other lines of inquiry will have to furnish the data necessary for decisive determinations.

Although the æther has been likened above to a metal at absolute zero, it is different inasmuch as it probably always contains as much electrokinetic energy as electrostatic. If the two electrons are rotating round their centre of inertia with linear velocities $u$, their total electrostatic energy can be immediately written down and their total electrokinetic energy obtained according to Heaviside (Phil. Mag. [5] xxvii.). First, for the electrokinetic energy of each electron due to its own translatory motion we have $e^2u^2/3V^2\alpha$. Again, if each electron has an angular velocity of rotation $\omega$ about an axis, its kinetic energy will be $a^2\omega^2I/3$. The potential energy of the electricity of an electron is $ee/2a$. Thus, then, the self-energy of the two electrons is

$$2e^2u^2/3V^2\alpha + 2a^2\omega^2I/3 + e^2/\alpha \ldots \ldots (33)$$

For their mutual kinetic energy we have $e^2u^2/V^2r$; if $r$ is the distance between their centres, and their mutual potential energy is $-e^2/r$, so that the total mutual energy is

$$e^2u^2/V^2r - e^2/r \ldots \ldots . (34)$$

According to the investigations of Thomson, Heaviside, Searle, and Abraham the formulæ for the electrokinetic energy hold only when $u/V$ is small. For larger values of $u/V$ the kinetic energy is no longer given by half the product of an inertia and the square of the velocity. It seems to me, however, that there is a promising line of research in assuming that for all values of $u/V$ kinetic energy is given by the expressions used above, and in deducing what modifications are required in the fundamental laws of electromagnetism to bring them into harmony with the principle that electric kinetic energy is always the product of the
square of the velocity and half the constant inertia. Thus in (34), when \( u = V \) the total mutual energy is nil. The kinetic and potential parts of the self-energy become equal if
\[
\alpha^2 \omega^2 t = \frac{u^2}{2} V^2 \alpha, \quad \text{i.e., if} \quad \alpha^2 \omega^2 = \frac{3u^2}{4}. \quad (35)
\]

The velocity of light then is such that if possessed by the electrons of a neutron it would make their mutual energy nil, and, subject to (35), would make their total energy consist of two equal parts, kinetic and potential. I have shown in "The Electric Origin of Molecular Attraction" that the energy of an electrostatic field, according to the neutron theory of the aether, is stored in the aether half as kinetic and half as potential energy. It would seem then as though the velocity of light through the aether is connected with the velocity of its electrons in much the same way as the velocity of sound through a perfect gas is related to the translatory mean velocity of its molecules, a possibility contemplated by the founders of the kinetic theory of gases, with the aether a gas.

One other point demands immediate attention. According to the electromagnetic theory the velocity of light is \((K\mu)^{-\frac{1}{2}}\), and much discussion has centred round the dimensions and the nature of \( K \) and \( \mu \). FitzGerald suggested (Phil. Mag. [5] xxvii.) that both \( K \) and \( \mu \) are the inverse of a velocity. Let us express this by putting
\[
K = \frac{c}{U}, \quad \mu = \frac{1}{cU'}, \ldots \ldots \text{in electrostatic units}
\]
\[
K = \frac{1}{cU}, \quad \mu = \frac{c}{U'} \ldots \ldots \text{in electromagnetic units} \quad (36)
\]

The ratio of the two units is \( c \), and for the free aether we have \( c = U = U' = V \). But in the aether of matter, by which we mean the aether enclosed by the smallest spheres circumscribing each atom, we cannot write \( U = U' \), but if \( c \) is the velocity of light through matter, we must have \( \sqrt{v^2} = UU' \). FitzGerald considered that possibly the velocities \( 1/K \) and \( 1/\mu \) are proportional to the square root of the mean turbulence of the aether. So we shall take \( U \) to be the velocity of the electrons in the aether. In the aether of matter the velocity of light is different from that in free aether, so that for it we cannot write \( c = U = V \). But for most substances \( \mu \) retains nearly the same value as in aether, so that \( U' \), even in the aether of matter, generally has a value close to that of \( U \) in free aether. We might provisionally regard \( U' \) as a velocity derived from the angular velocity of rotation of electrons round their own centres. In free aether it is equal to the translatory velocity \( U \) of the electrons, and in the aether of matter retains the same value as in free aether, because
velocities of rotation are not changed by the proximity of matter in the same way as those of translation. These stipulations then give us Maxwell’s law \( K = n^2 \). As is now well known, many substances like water have different values of \( K \), according to the circumstances of measurement, this phenomenon being connected with molecular structure and to be allowed for in the interpretation of Maxwell’s law. In “The Dielectric Capacity of Atoms” (Austr. Assoc. Adv. Sc. Jan. 1904) I have shown that values of \( K \) for the stuff of ions can be calculated from the ionic velocities, and that for the values found for many atoms the following law holds:

\[(K/n)^2B = \text{constant}, \ldots \ldots (37)\]

where \( B \) is the volume of the atom and \( n \) its valency.

This makes

\[n^2U^2/B = \text{constant}, \ldots \ldots (38)\]

When \( n = 1 \) we have \( U^2/B \) constant, and \( 1U^2/IB \) also constant; and as we assume \( I \) to be constant, we have the kinetic energy of translation of the aether in all monad atoms the same per unit volume. For atoms of higher valency it would appear that the effective inertia of the aether of the atom for translatory motion is proportional to \( n^2 \). This relation would establish a connexion between the aether of an atom and its valency, that is between the special doublets which confer its valency on an atom and the neutrons of the aether. This requires further investigation. It is noteworthy that the translatory kinetic energy of the neutrons in a monad atom is the same per unit volume in all atoms, just as in all gases at the same pressure and temperature the translatory kinetic energy per unit volume is the same. FitzGerald’s idea is substantially the same as Fresnel’s, who took the ratio of density of aether in matter to density of free aether, or \( \rho/\rho_e \), to be equal to the square of the index of refraction, which by Maxwell’s law becomes equal to \( K \). But if the translatory momentum of the neutrons per unit volume is the same in aether everywhere, then \( U\rho = V\rho_e \) and \( \rho/\rho_e = V/U = K \), which shows that FitzGerald’s principle brings us back to Fresnel’s important law. It should be noticed that in the foregoing we have taken account of three distinct ways in which kinetic energy can exist in the aether: first by rotation of an electron round its centre, connected with magnetic permeability of the aether; second by rotation of neutron, which is the same as translation of electron; and third by motion impressed on the doublets of matter by the atoms, this being the origin of radiation.

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