The inspection of these figures shows that when the dark space is in geometrical contact with the edge of a luminous source, it appears as a dark protuberance projecting from the surrounding dark space. With the receding of the dark space towards the interior of the luminous source, the connecting ligament becomes thinner, and finally disappears. The inner dark disk is, however, a little elongated, and assumes a pear-shaped appearance; while the external dark space bulges out towards the luminous source. The change is only transient; with further ingress the dark disk becomes circular, and the swelling of the external dark space vanishes. The accompanying diagram (fig. 12) will represent approximately the various stages of dark space within a luminous source, as observed by means of a telescope.

Thus, to the first approximation, we have arrived at the explanation of drop formation during the transit of inferior planets.

If the dark space be taken nearly equal to the luminous source and have small protuberances, we can, by similar process, obtain a result which has close analogy with Baily’s beads.

II. Relative Motion of the Earth and Æther.

By William Sutherland*.

The experiment of Michelson and Morley, described in the Philosophical Magazine [5] xxiv. p. 449, has created quite a dilemma in the physics of the æther; for while the great body of general evidence tends to show complete independence of the æther near the earth on the earth’s motion, this celebrated experiment has been supposed to prove defi-

* Communicated by the Author.
nitably that the earth's surface and the adjacent æther have no relative motion. I propose now to show how a slight alteration in the point of view of the theory of that experiment will make it appear that, until a special adjustment for sensitivity of the optical apparatus has been made, it is not competent to decide as to the relative rest or motion of earth and æther.

For the sake of clearness let us briefly repeat the authors' account of the theory of their experiment along with their diagram. \( b \) and \( e \) are two mirrors at right angles to one another (fig. 1), and at equal distances \( D \) from \( a \) a piece of glass inclined at \( \pi/4 \) to them, and intended to divide a beam of light \( sa \) into reflected and transmitted parts going to \( b \) and \( e \) respectively. Suppose the whole apparatus to be moving in the direction \( sc \) with velocity \( v \) relative to the æther in which the beam of light is moving with velocity \( V \); then while the reflected beam is going to \( b \) and back \( a \) is moving to \( a_1 \), so that the path of the reflected part is \( aba_1 \), while that of the transmitted part is \( ace_1 \); at \( a_1 \) the former is partly transmitted and the latter partly reflected to the telescope under conditions favourable to the occurrence of interference.

Along \( ac \) the beam moves with velocity \( V - v \) relative to \( c \), so that the time of traversing \( ac \) is \( D/(V-v) \); similarly the time for \( ca_1 \) is \( D/(V+v) \), and hence

\[
ac + ca_1 = DV \left( \frac{1}{V-v} + \frac{1}{V+v} \right) = \frac{2D}{1-v^2/V^2}.
\]

The path of the other beam is taken to be \( aba_1 \), whose length
is evidently $2D(1 + \frac{v^2}{V^2})^4$, so that with omission of the third and higher powers of $\frac{v}{V}$ the difference between $aca_1$ and $aba_1$ can be set down as $D\frac{v^2}{V^2}$. If $v$ is the velocity of the earth in its orbit, then if the whole apparatus is turned through $\pi/2$ the longer path becomes the shorter, and vice versa: so that the difference of path which occasioned the interference-fringes in the first position is altered in the second by $2D\frac{v^2}{V^2}$, which ought to produce a difference in the position of the fringes in the telescope. And this motion of the fringes is what Michelson and Morley found not to occur, even after they had given the apparatus a sensitiveness which they supposed to be quite adequate to its accurate measurement, minute as is $\frac{v^2}{V^2}$, namely about $10^{-8}$. Hence the startling conclusion that the relative motion of earth and æther must be small compared to the earth's orbital velocity, in short that the earth drags the æther with it.

In the supplement to their paper Michelson and Morley (p. 460) show that the angle which $ab$ makes with the mirror at $b$ is more strictly $\frac{v}{V} + \frac{v^2}{2V^2}$, while $ca_1$ after reflexion makes an angle with the normal to $b$ which is more strictly $\frac{v}{V} - \frac{v^2}{2V^2}$, so that the two rays which are to interfere after passing $a_1$ are inclined at an angle $\frac{v^2}{V^2}$ to one another. This is of no importance in the actual experiments, because to get interference-bands of a convenient width it is necessary to have the rays inclined at a much larger angle than this, though still at a very small angle, which is obtained by slight derangements of the mirrors and plate from the ideal mathematical positions assigned to them in the theory of the experiment.

To ascertain what really happens with the two interfering rays, let us take a small area of plane wave equal to the area of the pupil of the observer's eye, and take its image in $b$ as it appears immediately after reflexion at $a$, and let us take the image of its transmitted part after reflexion at $c$ and $a_1$; then if the whole apparatus and æther were at rest, and the angles were all exactly as supposed, we should have the two images coincident as represented in section at AB (fig. 2).

Now suppose that by slight derangement the images are separated as in CD and EF. Fringes now appear whose width depends on the angle COE. When the images coincided they represented coincident trains of waves with double illumination along their path. If the derangement, which separated the image AB into two, left them still parallel, there would be only circular fringes visible in any plane parallel to them, and only a uniform resultant central illumination to an infinitely small eye moving along a normal,
which will have the special value zero when the distance
between the separated images is equal to half a wave-length

![Diagram](image)

...of the light employed. Thus, then, while the images are
parallel no straight fringes can be seen, and the effect of
varying the distance between the images is to cause motion
among the circular fringes. Returning now to the case of
a symmetrical angular separation of the images as at CD and
EF, we see that the central bright fringe (really dark in the
actual experiments because of the opposite conditions of re-
flexion of the two beams) will be seen anywhere along OP;
and the next may be said to be approximately along the line
GH (and only approximately straight), at such a distance
from O that GH is equal to \( \lambda \), the wave-length. Thus the
interval between the fringes PQ is approximately given by the
equation

\[
PQ = \frac{s}{\lambda/2 \tan \theta_A}.
\]

For brevity we will confine the discussion to two-dimensioned
space, that is, to straight-line waves travelling in a plane.

Next let the relative motion of the aether and the apparatus
cause the separation of the images contemplated by Michelson
and Morley, namely by a distance \( s = Dv^2/V^2 \), which is also
accompanied by the angular rotation of the two images through
\( v^2/2V^2 \) in opposite directions; but when COA is large com-
pared to \( v^2/V^2 \) this last effect can be neglected. Suppose,
then, that CD is moved to C'D' so that O'O' = s, and let C'D'
cut EF in L. It seems to be assumed by Michelson and
Morley that the locus of the central fringe moves along to
LR, so that

\[
OM = PR = LM/\tan \theta_A = s/2 \tan \theta_A,
\]

and therefore that the whole system of fringes is moved later-
ally by the same fraction of the width of a fringe as \( s \) is of \( \lambda \).
Now this can be true only if the images are symmetrical with respect to \( L \), or if each wave-front is absolutely uniform in character. But in dealing with wave-fronts of light in connexion with interference we must be careful to conjoin the effects of parts only of identical origin, as, for instance, that of \( C \) with that of \( E \), or that of \( O' \) with that of \( O \), or that of \( L \) in \( C'D' \) with that of \( L \) in \( EF \).

But in the actual experiments it is practically impossible to secure that the two images intersect in a point that corresponds to itself as \( L \) does. In general we must assume that the two images \( C'D' \) and \( EF \) intersect under the conditions represented in the next diagram (fig. 3), where \( O \) in \( EF \) and \( O' \) in \( C'D' \) are images of the same point in the original wave-front.

Let \( OO'=c \); then if \( T \) is any point at distance \( x \) from \( O \) in \( EF \), the corresponding point \( U \) in \( C'D' \) is at distance \( c+x \) from \( O \). Let us bisect \( FOD' \) by \( AB \), and find the value of \( x \) which determines a pair of corresponding points so that they are equidistant from a point whose polar coordinates relative to \( O \) and \( OB \) are \( r \) and \( \theta \). Denote the angle \( FOB \) by \( \alpha \); then the equality of \( PT \) and \( PU \) gives

\[
r^2 + x^2 - 2rx \cos(\theta + \alpha) = r^2 + (c + x)^2 - 2r(c + x) \cos(\theta - \alpha),
\]

\[
\therefore 2x(c - 2r \sin \theta \sin \alpha) - 2r \cos(\theta - \alpha) + c^2 = 0;
\]

and treating \( \alpha \) as a small angle

\[
2x(c - 2r \alpha \sin \theta) - 2rc \cos \theta - 2rc \sin \theta + c^2 = 0 \text{ nearly.}
\]

Denote \( r \cos \theta \) by \( q \), and \( r \sin \theta \) by \( p \), and then

\[
2x(c - 2p \alpha) + c(c - 2p \alpha) - 2cq = 0.
\]

In this equation first suppose that

\[
e - 2p \alpha = 0,
\]

then either \( c = 0 \) or \( q = 0 \). If \( c \) happens to be zero, then
either $p=0$ or $\alpha=0$. The case of $p=0$ is of no interest, and with $c=0$, $\alpha=0$ we have the case of absolutely coincident images with no fringes at all. Thus the case with $c-2\rho \alpha=0$ and $c=0$ can be dismissed, and we have to consider next $c-2\rho \alpha=0$ and $q=0$, which amounts to this, that at the particular position of $P$ given by $p=c/2\alpha$ and $q=0$, both of each pair of corresponding points are nearly equidistant from $P$, so that both of each pair of corresponding disturbances reach this position of $P$ in the same phase. In this case there is one point $p=c/2\alpha$ and $q=0$ which stands nearly in the same sort of symmetry with respect to the two images as $O$ does when $O'$ and $O$ are identical. We will return to this as a special case after we have studied the general case in which $c-2\rho \alpha$ is not equal to zero. Here we have

$$x = \frac{c\{q-(c-2\rho \alpha)/2\}}{c-2\rho \alpha}.$$  

Let the state of affairs we have been discussing so far be that in which there is no relative motion of the apparatus and the æther, so that the want of coincidence of the two images is due entirely to experimental imperfection; and now suppose the apparatus to acquire its velocity $v$ relative to the æther, the effect of which is to shift $C'D'$ relatively to $EF$ in the manner contemplated by Michelson and Morley. Let us suppose the shift to be the simplest possible, namely, that of $C'D'$ parallel to itself to $GH$ through a distance $OK = s = Dv/\sqrt{\gamma}$ along the normal to $AB$, and let $GH$ intersect $EF$ in $L$, which is now to be regarded as a new origin. $c$ has not been altered by the shift, $p$ has been increased by $OY = s/2$, and $q$ has been diminished by $YL = s/2\tan \alpha = s/2\alpha$ nearly; thus then for the distance $x'$ defining the distance of a point along $LF$ from $L$ which is at the same distance from $P$ as its corresponding point in $LH$, we get by making in (3) the changes indicated

$$x' = \frac{c\{q-s/2\alpha-(c-2\rho \alpha-s\alpha)/2\}}{c-2\rho \alpha-s\alpha}.$$  

so that approximately as $s$ and $\alpha$ are small

$$x-x' = \frac{c}{c-2\rho \alpha-s\alpha} \cdot s.$$  

But $x'$ is measured from $L$, so that the actual shift of the corresponding points is $x-x' = OL = x-x'-s/2\alpha$

$$= \frac{s}{2\alpha} \cdot \frac{2\rho \alpha}{c-2\rho \alpha}.$$  

Therefore if \( c \) is large compared to \( 2p\alpha \) the shift of corresponding points is only a small fraction of that \((s/2\alpha)\) contemplated by Michelson and Morley for the fringes. Now to see how the consideration of corresponding points bears upon the problem we have only to remember that each wave-front propagates itself normal to itself, and that therefore the best interference effects are to be sought along the normal to \( AB \) which is equally inclined to the normals to the two image waves; if we fix our attention on two small elements of the image waves around two corresponding points, we see that their most vivid interference effects will be along the normal to \( AB \) which is at distance \((x+c/2)\cos\alpha\), or nearly \( x+c/2 \) along \( AB \) from \( O \). In order that \( q \), which is \( x(c-2p\alpha)/c + (c-2p\alpha)/2 \), should be nearly this \( x+c/2 \), the fraction \( 2p\alpha/c \) is to be small, or \( c \) is to be large compared to \( 2p\alpha \), which is the condition just found in order that the shift of corresponding points should be small compared to that of the fringes sought by Michelson and Morley. If the eye of the observer is placed in \( OZ \) (fig. 3), that is if \( q=0 \), then \( x=-c/2 \), and in the whole system of possible fringes there is along \( OZ \) a peculiar central region characterized by a certain symmetry; but at the same time to an eye placed anywhere the central band in its field may be characterized by maximum clearness, yet it will have none of the peculiar character of the one absolutely symmetrical central one. To an eye placed anywhere the central band in its field is at such a point that the mean distance from equal corresponding areas of the two images is the same; and according to what we have just seen if the one image is shifted through a distance \( s \) along the normal to the mean position of the two images, then the position of corresponding points is only changed by the small fraction \( 2p\alpha/(c-2p\alpha) \) or \( 2p\alpha/c \) of \( s/2\alpha \), and therefore the fringes will be seen in the eye to move approximately only the small fraction \( 2p\alpha/c \) of the expected \( s/2\alpha \). If things were adjusted so that the absolutely central region of the fringes appeared in the eye, then just as in fig. 3 \( O \) moves to \( L \), the complete shift \( s/2\alpha \) of the central region would be observed; but this adjustment would be a tedious business.

In the final experimental arrangements the simplicity of the scheme given in fig. 1 was somewhat departed from, because each half of the divided beam was reflected backwards and forwards four times along its initial path in order to increase the effective value of \( D \); the images were brought into apparent coincidence and so adjusted that \( 2\alpha \) was such as gave fringes of convenient width; but \( c \) was quite unknown and might be large compared to \( 2p\alpha \), or say \( 2D\alpha \), without
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detection, because while \( \alpha \) was adjusted for by reason of the necessity of making the fringes of convenient width, there was no adjustment for making \( c \) zero, beyond the comparatively rough one of causing images of an object to coincide apparently, and therefore on the average \( c \) may be assumed to be large compared to \( 2 \alpha \). The case in which \( c \) becomes not very different from \( 2 \alpha \) has already been partly discussed in the extreme form when \( c = 2 \alpha \), a relation which we saw makes \( x \) indeterminate. When \( c - 2 \alpha \) is small but not zero, since 
\[
x = -\frac{c}{2} + \frac{qc}{(c - 2 \alpha)}
\]
we can, by keeping \( q \) small, also keep \( x \) as near \( -c/2 \) as we please, so that the central region of the fringes is still peculiar and characteristic. In this case the approximation given for \( x' - x \) in (5) is no longer of use; and we must reason in the following way, that before the shift (fig. 3), the central region will lie along \( OZ \), and after the shift along a parallel to \( OZ \) through \( L \), so that the system of fringes shifts as expected by Michelson and Morley; in short, when \( q \) is small we are observing near the central fringe which moves the full \( s/2 \alpha \). Thus we see that while an improper use of the formula (6) might make it seem as if we could get infinite magnification of the effect looked for in the experiment, in reality the shift \( s/2 \alpha \) of the fringes is the largest practically obtainable, and can be secured only by making either \( q \) nearly equal to 0, or as we saw before (fig. 2) \( c = 0 \); while if \( c \) is allowed to be larger than \( 2 \alpha \) the shift may be any fraction of this maximum. In other words, the shift expected by the experimenters can be obtained in only two cases: first when the intersection of the two images corresponds to itself, in which case the eye may observe anywhere; and second, when there is a lateral shift of one image relative to the other, and the eye is in the axis of quasi-symmetry \( OZ \). If the contention here advanced is sound, it appears that the failure of Michelson and Morley to get evidence of the relative motion of earth and æther is due to the absence of a certain adjustment required to give their method the sensitivity aimed at, and not to any real defect in the theory of the experiment. Their method has been since applied to other attempts to measure motion of the æther, as in Lodge's experiment with the whirling steel disks (Phil. Trans. clxxxiv. 1893), Threlfall and Pollock's on the Effect of Röntgen Rays (Phil. Mag. [5] xlii.), and Henderson and Henry's (Phil. Mag. [5] xliv.) on the Motion of æther in an Electromagnetic Field; in all these experiments only negative results have been obtained. But it is to be remembered that in the last three the two parts of the divided beam are sent in opposite directions round the same path, whereas in Michelson and Morley's experiment the two beams travel in independent
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paths: now in the former case if the two beams travel D between separating and meeting, and meet again at an angle 2α, then the linear separation of corresponding points may be taken as 2AD, which amounts to the same as our 2αp, and hence the conditions are such as favour the degree of sensitiveness expected by Michelson and Morley; while in their own experiment, as each of the separated beams is reflected fifteen times in its own independent path there is opportunity for a considerable lateral shift of the one beam relative to the other when they meet, although both are adjusted as nearly to parallelism as is necessary, that is to say that c is independent of 2αp, and the sensitiveness of the system of fringes is unknown, but in all probability small compared to that expected. In their celebrated repetition of Fizeau's great experiment on the effect of running water on the æther Michelson and Morley got their well-known positive result, but in this case the divided beams were sent in opposite directions round the same path, so that the optical arrangement had the sensitiveness expected. It is the use of multiple reflexion along different paths in the experiment on the relative motion of earth and æther that introduces the possibility of comparatively large lateral shift.

If the argument in this paper is correct it ought to be possible by careful adjustment for the requisite smallness of c, or for getting the absolute central band into the field of view, to give the Michelson and Morley apparatus the sensitiveness desired for measurement of the relative motion of earth and æther; and in any case an experimental examination of the effect of lateral shift seems desirable.

Melbourne, Sept. 1897.

III. Transmission of Radiant Heat by Gases at Varying Pressures. By Charles F. Brush*

Before describing my own investigations on the transmission of heat by gases, I shall refer briefly to the classical work of a somewhat similar nature by MM. Dulong and Petit early in the present century. Their memoir entitled "Researches on the Measure of Temperatures, and on the Laws of the Communication of Heat," gained the prize voted by the Academy of Sciences in 1818. A translation of this important paper may be found in the "Annals of Philosophy," for February, March, April, and May, 1819.

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