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**XLIX. Thermal Transpiration and Radiometer Motion.**

*By William Sutherland*. 

[Continued from p. 391.]

**PART II.—Radiometer Motion.**

**REYNOLDS**, in treating of radiometer motion in connexion with his discovery of thermal transpiration, showed that fundamentally both phenomena are traceable to the same general cause: the object of the rest of this paper is to bring out this fact more clearly, and to establish theoretically the general laws of radiometer motion for comparison with the experimental results of Crookes and Pringsheim.

In the theory of thermal transpiration, we have seen that under suitable conditions the variation of temperature along a passage through a porous plate can produce a certain difference between the pressures at its hot and cold ends, and therefore the solid wall of the passage must be exerting a tangential force \( F \) from cold to hot, such that, \( R \) being the mean radius of the passage,

\[
F = \pi R^2 (p_2 - p_1),
\]

and, accordingly, the gas in the passage exerts a tangential

* Communicated by the Author.
force $F$ on the solid from hot to cold. If then a porous plate
had one face heated and was hung on to a string with this
face and the opposite cool one vertical, the tangential force
$F$ acting along all the passages would deflect the string from
the vertical, a case of radiometer motion; if two such plates
were mounted in a vertical plane and free to revolve round a
vertical axis lying between them, and one face of one was
warmed by irradiation, it would move away from the source,
and thus a continuous rotation could be kept up as in an
ordinary radiometer.

In discussing thermal transpiration we confined our atten-
tion to fine tubes, such as might represent the passages in
porous plates; but as we saw that the phenomena depended
for the most part on the ratio of the radius of a passage to the
mean free path of the gas, it follows that our deductions for
fine tubes will hold for tubes of any size with rare enough
gas to give a free path as large as may be necessary; thus
with the means of getting high enough vacua and with
delicate manometers it should be possible to demonstrate
thermal transpiration along an ordinary gas-pipe or the
largest gas main; in the radiometer we have generally to do
with thermal transpiration going on in spaces of ordinary
size.

We have already obtained in (5) an expression for the
traction exercised on the gas in the tube by the whole surface
of a tube along which the temperature varies; thus

$$F = nmu^2\pi R^2;$$

a curious result that the traction on the surface should be
proportional to the square of the radius, but it is to be
remembered that the tube is supposed to be long enough in
comparison with its diameter, and of sufficient thermal
capacity, to dominate the temperature of the gas so thoroughly
that the temperature throughout any section of the tube is the
same as that of the wall. This traction has been found for
the case when the motion due to thermal transpiration along
the tube has become steady; but in connexion with radiometer
motion it is necessary to consider the traction before the
steady state is established. Imagine a solid surface over
which the temperature varies to be suddenly introduced into
a mass of gas at rest and uniform in temperature, and let us
determine the traction which the solid immediately exerts on
the gas. The first effect is to make the layer of gas in contact
with the solid take the temperature of the solid at every point
of the surface, and therefore each molecule that encounters
the surface acquires on the average the velocity $u$ given by
(4), and the number that encounter unit surface in unit time being $nv/4$, the total momentum imparted to the gas in unit time by a surface $S$ is given by

$$F = Snmvu/4.$$  

(20 a)

This is the initial value of the traction; but as the velocity $u$ is carried out to the remoter parts of the gas, a molecule which encounters the surface having come from a region where it had already acquired a fraction of $u$ does not receive the whole of $u$ from the solid, and therefore the traction diminishes with time. To determine the final value when the motion of the whole gas is steady we may consider the simple case of two parallel planes the variations of temperature over which are such as to produce velocities $u_1$ and $u_2$ in a fixed direction in the gas in contact with the two surfaces; then in the steady state we may suppose the transition from $u_1$ to $u_2$ to occur linearly, so that the velocity $u$ at distance $x$ from one of the planes is $u_1 - (u_1 - u_2)x/D$, where $D$ is the distance between the planes; then the mass of gas that flows in unit time along any layer of width $b$ and thickness $dx$ is $nmbu dx$, and the momentum imparted in unit time to the layer is $nmbu^2 dx$, and therefore the total momentum acquired by the gas between the planes is

$$\int_0^D nmbu^2 dx = nmbD(u_1^2 + u_1u_2 + u_2^2)/3.$$ 

Obviously the planes impart the respective fractions $u_1^2/(u_1^2 + u_2^2)$ and $u_2^2/(u_1^2 + u_2^2)$ of this, so that the traction per unit area of the first plane, if its length in the direction of motion is $l$, is

$$nmDu_1^2(u_1^2 + u_1u_2 + u_2^2)$$ 

$$3l(u_1^2 + u_2^2);$$

but it is really a useless artificiality to consider the traction per unit surface, as most of the traction is really exerted on the gas near its entrance to the space between the planes, and we will therefore confine our attention to the total tractions. As before in the case of the tube, the result that the traction should be proportional to the sectional area between the plane is peculiar, but it is true only when the planes dominate the temperature of the gas in such a manner that $u$ is a linear function of the distance from either. Thus the initial total traction on the first plane is $Snmvu/4$, which is proportional to the surface $S$, that is to both width and length, but independent of distance from neighbouring surfaces; and the final traction in the steady state is

$$nmbD(u_1^2 + u_1u_2 + u_2^2)u_1^2/(u_1^2 + u_2^2);$$
which is independent of the length but proportional to width
and distance from neighbouring surface. To bring out the
full signification of these we had better introduce the value
of \( u \), and let us suppose \( u' \) to be 0; then the initial and final
total tractions exerted by a plane of varying temperature at
distance \( D \) from a plane of constant temperature are

\[
\begin{align*}
-\frac{Snmv^2}{24}(n'/n + v'/v), \\
\delta m Dv^2\frac{\lambda^2(n'/n + v'/v)^2}{108}.
\end{align*}
\]

When the conditions are such that the pressure can remain
constant between the surfaces, \( n'/n + 2v'/v = 0 \), and then these
become \( Snmv\lambda v'/24 \) and \( \delta m Dv^2\lambda^2v^2/108 \), both acting from cold
to hot, and therefore the equal and opposite reaction of gas
on the surface is from hot to cold.

But when the conditions are not such that the pressure is
kept constant, but that a difference of pressure is established
by thermal transpiration which goes on till a steady state is
established, the effect of the difference of pressure may be
much greater than that of the traction, as the following
example will show:—A piston is inserted into a cylinder which
it does not quite fit, and is fixed immovable so as to leave a
clear space of sectional area \( a \) between itself and the cylinder,
and the cylinder is closed; when one end is heated a fall of
temperature gets established along the cylinder, and the gas
at the cold end begins to transpire through the narrow space
into the hot end until the difference of pressure \( p_2 - p_1 \)
sufficient to stop the flow is established; then the total traction
of gas on the side of the piston is \( a(p_2 - p_1) \), while on the area
\( A \) of the hot end of the piston there is a total pressure
\( A(p_2 - p_1) \) in excess of that acting on the cold end, so that the
total force urging the piston from hot to cold is \( (A + a)(p_2 - p_1) \),
which may be made as much more important than the total
traction as we please by diminishing \( a \). If the piston is freed
it will begin to move under the force \( (A + a)(p_2 - p_1) \), and
become an exaggerated instance of radiometer motion.

This example makes clear the lines on which to formulate
a general theory of radiometer motion; we have only to
adapt our transpiration formulæ established for circular tubes
to the case of any space bounded by solid walls over which the
temperature varies. In the general problem of radiometer
motion we have to do with a solid surface over which the
temperature varies and which therefore is subject to a traction
from hot to cold, and also establishes a higher pressure in the
gas towards its hot end than at the cool end; the relative
importance of traction and difference of pressure in producing
motion of the body to which the surface belongs depends
entirely on the relative position of the movable surface and the fixed surfaces surrounding it. We have seen that for a tube the total traction is $nm\nu^2\pi R^2$, and for a passage bounded by two parallel planes of width $b$ and distance $D$ apart with the same variation of temperature along both, it is $nm\nu^2bD/2$ along each plane, and as $\pi R$ corresponds to $b$ and $R$ to $D/2$ we see that the traction exerted between the walls of a cylinder of any section and the contained gas may be written in the form $nm\nu^2sD/2$, where $s$ is the perimeter of a right section of the cylinder, and $D$ is a mean value of the distance between opposite parts of the perimeter. For the difference of pressure established by thermal transpiration along a tube of any section we may use the equations (14) and (17) if in them we interpret $2R$ as a mean value of the distance between opposite parts of the perimeter; and in the case where there is a variation of pressure across only a fraction of the perimeter, as for instance in the case where one plane wall has a varying temperature and the opposite one a uniform temperature, we must multiply by a fraction not greatly different from that fraction ($1/3$ instead of $1/2$ in our example). We can therefore state the fundamental equations of radiometer motion as follows:—If across a length $b$ of the perimeter $s$ of any cylinder a variation of temperature is suddenly established whose average rate is $\nu'$ over a length $l$, then the initial total traction between solid and gas is approximately

$$blmm\nu'v'/24.$$  (22)

When the steady state is reached the total traction is approximately

$$bDnm\nu^2v^2(n'/n + \nu'/\nu)^2/108,$$

or $bDnm\nu^2v^2(p'/p - \nu'/\nu)^2/108$, (23)

and the difference of pressure between the two ends of $l$ is

$$p_2 - p_1 = \frac{b}{s} \frac{v_2 - v_1}{v_3 + v_1} \frac{1}{A'(p_2 + p_1)/4 + B'/2 + 1/(p_2 + p_1)},$$  (24)

where the values of $A'$ and $B'$ are those given in (19), with $R_2 = R_1 = R$. No proof has been furnished here that the introduction of the fraction $b/s$ rigorously adapts our expression (19) to the case where only a fraction $b/s$ of the boundary is operative in producing thermal transpiration, but it is a reasonable enough approximation for experimental results at present available, closer approximation could easily be calculated if required. If in (23) we write $(p_2 - p_1)/l$ and $(v_2 - v_1)/l$ for $p'$ and $\nu'$ we can express the total traction in the steady state entirely in terms of $v_2$ and $v_1$, which completes the solution.
With these results we can now state what ought to be the behaviour of a radiometer, and as Crookes and Pringsheim found the best form of instrument for investigating the laws of the radiometer experimentally to be one in which a single vane of mica blackened on one side was attached with its planes vertical to a horizontal arm attached to a vertical torsion fibre, the whole being suspended in a glass bulb capable of being filled with any gas at any pressure, we will discuss the theoretical laws of such a form. Let $D$ be the mean distance of the edge of the vane from the glass wall immediately opposite it, $b$ the perimeter of the vane, $s - b$ the perimeter of the glass wall opposite, $E$ the area of each face of the vane, $E + S$ the sectional area of the bulb ($S$ being small compared to $E$) in the plane of the vane, and $\beta$ the thickness of the vane; when the black face is irradiated let its temperature become $\theta_2$, that of the clear face being $\theta_1$, then there is a fall of temperature $\theta_2 - \theta_1$ through the thickness of the vane, and thus the thickness of the vane becomes a surface capable, along with the surface of the bulb opposite it, of starting thermal transpiration from the cold edge to the hot, with elevation of the pressure in front of the hot face to $p_2$, and depression of that behind the cold face to $p_1$; when a steady state is established the total traction on the surface of varying temperature must be approximately equal to $S(p_2 - p_1)b/s$, and the excess of total pressure on the black face over that on the clear face is $E(p_2 - p_1)b/s$, so that the total force deflecting the vane whose moment is to be balanced by the torsion couple of the fibre is $(E + S)(p_2 - p_1)b/s$. Thus the total deflecting force is

$$(E + S) \frac{b}{s} \frac{v_2 - v_1}{v_2 + v_1} + \frac{1}{9D^2\psi_0^2(p_2 + p_1)} + \frac{1}{16\psi_0^2(v_2 + v_1)^4} + \frac{1}{\psi_0(v_2 + v_1)^6} + \frac{1}{p_2 + p_1}$$

or

$c/\{A'p + B' + 1/p\}$, where $p$ is the mean pressure $(p_2 + p_1)/2$. This equation contains all the theoretical laws of radiometer motion when $S$ is very small compared with $E$. If everything is kept constant except the mean pressure $(p_2 + p_1)/2$ there is a value of the mean pressure of the gas in a radiometer for which the deflecting force is a maximum, a very important point in radiometer construction. When the pressure is high enough the last two terms in the denominator may be neglected and the deflecting force is inversely proportional to the pressure, and when the maximum is passed and the pressure becomes small enough the first two terms may be neglected and the deflecting force becomes proportional to the density.
Mr. W. Sutherland on Thermal

and dies away indefinitely with increasing exhaustion of the bulb.

It will be seen that the deflecting force depends on the dimensions of the apparatus in a somewhat complicated manner, but that the most important principle is that (as regards the denominator) it increases with diminishing distance $D$ between vane and glass wall, except at pressures so low that the first two terms are negligible; and as diminishing $D$ means in general an increasing value of $b/s$, we see that in general at all pressures the efficiency of the radiometer is increased by bringing the edge of the vane nearer to the glass wall. Other things being equal the deflecting force is proportional to the total sectional area $E+S$ of the bulb.

As regards the effect of the nature of the gas on radiometer motion the equation shows that at pressures low enough for neglecting the first two terms of the denominator all gases give the same deflecting force, a theoretically interesting result, but not of much practical importance: the practically important matter is to determine how the different gases compare, each at its maximum effectiveness; now when the deflecting force is a maximum $p_2 + p_1 = 4\eta_0(v_2 + v_1)^2/3Dv_0$, or $D = 2\lambda$, where $\lambda$ is the mean path at $(p_2 + p_1)/2$, and the deflecting force becomes proportional to $4\eta_0(v_2 + v_1)^2/3Dv_0$, so that the most effective gas is that for which $\eta_0(v_2 + v_1)^2/v_0$ is largest, that is to say, for which $\eta_0/m^b$ is largest; compare for instance hydrogen and oxygen, $\eta$ for $H_2$ is ‘44 of that for $O_2$, while $m^b$ is 1/4, and thus at the pressure of maximum efficiency $H_2$ is 1·76 times as efficient as $O_2$, and at higher pressures the advantage of $H_2$ increases till its efficiency is 1·76$^2$ that of $O_2$.

From Rayleigh's measurement of the viscosity of helium as ‘96 of that of air (Proc. Roy. Soc. Jan. 1896) while hydrogen's is about ‘5, and with 4 as the molecular mass of helium as against 2 for hydrogen, it would appear that helium ought to be nearly 2$^b$ or 1·4 times as efficient in a radiometer as hydrogen.

The equation (25) contains the laws of the dependence of radiometer motion on the temperatures of the faces of the vanes, although as these temperatures have never been measured experimentally, we cannot verify them as they stand; but to a certain extent we can bring them within the range of experimental verification in the following manner. When the black face of a vane is suddenly irradiated the temperature of the black face suddenly rises, while that of the clear face is unaltered, and the fall of temperature is confined for the first moment or two to the thickness of the layer of lampblack; the first deflexion of the vane takes place in
Transpiration and Radiometer Motion.

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accordance with these conditions, but if the vane is steadily irradiated, conductivity soon establishes a steady distribution of temperature through the vane from front to back with permanent temperatures \(\theta_2\) and \(\theta_1\), so that the first deflexion alters until it attains the fixed value due to these steady conditions. Suppose the black face to be irradiated by a candle, and let \(Q\) be the amount of heat it absorbs per unit area per unit time, and \(E\) the corresponding amount emitted by the clear face; then if we ignore loss of heat by the edge, and denote the conductivity of the substance of the vane by \(k\),

\[Q = k(\theta_2 - \theta_1)/\beta = E.\]  

Now in (25), \((v_2 - v_1)/(v_2 + v_1)\) may be written

\[\frac{(v_2 - v_1)^2}{(v_2 + v_1)^2},\]

or

\[\frac{(\theta_2 - \theta_1)^2}{(\theta_2^2 + \theta_1^2)^2},\]

in which the denominator is nearly \(4(\theta_2 + \theta_1)\), since the difference of \(\theta_2\) and \(\theta_1\) is small; and as in the experiments \((\theta_2 + \theta_1)/2\) was always an ordinary temperature it may be taken as constant, so that \((v_2 - v_1)/(v_2 + v_1)\) was always closely proportional to \(\theta_2 - \theta_1\) and therefore to \(Q\beta/k\); but \(Q\) is inversely as the square of the distance of the candle from the vane and directly as the number of candles at that distance (the candles being as nearly as possible in the normal to the centre of the vane), thus the deflecting force varies as the number of candles and inversely as the square of their distance from the vane, which is the experimental result (Crookes, 'Nature,' xiii.); the deflecting force is proportional to the thickness of the vane and inversely proportional to the conductivity of its material, hence the advantage of using a substance such as mica for the vane, and the disadvantage of using metal.

To show that this theory of the radiometer is in harmony with the experimental facts, we will briefly describe the general results of Crookes's numerous experiments, and it will be seen that they accord with the deductions from our formulæ.

Crookes obtained his most valuable quantitative results with an apparatus such as the one of which we have just considered the theory. The bulb was continued into a vertical tube for containing a torsion fibre nearly a metre long, and the rectangular plate of roasted mica was attached directly to the fibre so that its plane was vertical and its centre at the centre of the bulb; a continuation of the line of the fibre divided one face of the plate into two equal halves,
one of which was lamplblack. For all the experimental niceties reference must be made to the original paper (Phil. Trans. clxxii.)

It is obvious from the description of this apparatus that it does not comply with the conditions under which (25) was established, as the mica plate is probably only a fraction of a millimetre in thickness and between 5 and 10 millim. from the glass bulb where it is nearest, so that the length of the region in which thermal transpiration occurs is much less than its width, whereas in (25) the contrary is supposed to be the case. The chief effect of the difference in these conditions will be that thermal transpiration, instead of going on over the whole distance between edge of plate and bulb, will extend to a distance from the edge of the plate which will depend on the conductivity of the gas; in fact, if we move along the shortest distance between plate and bulb we shall find the fall of temperature across that line grow less as we leave the plate and become negligible before we reach the bulb; but the better the conductivity of the gas the farther will the dominating influence of the edge of the plate extend; therefore in our formulae, when applied to Crookes's experiments with the torsion radiometer, $D$ must be interpreted as a function of conductivity $k_\prime$. Then $b$ being the length of the edge of the black half of the plate, the area $S$ over which thermal transpiration is effective may be taken to be $bD$, over which at the front and the back of the plate there is an average difference of pressure $p_2 - p_1$, which, however, will not be maintained over the whole front and back of the plate, because there is so much facility of escape for the gas, but only near the edge, so that probably $E$ varies as $bD$; thus $(E + S)b/s$ will be replaced by $bK$, where $K$ is a function of $k_\prime$. Another effect of the fact that thermal transpiration occurs only to a certain distance from the edge of the plate will be to reduce the effect of slipping, seeing that the velocity of transpiration dies away to zero in the gas. To indicate that slipping has not its full theoretical effect we had better change $B'$ to $B''$, and to remind ourselves that in $A'$ and $B'$ the symbol $D$ or $2R$ now means a function of $k_\prime$, we will change $A'$ to $A''$ and $B''$ to $B'''$ and put

$$bK(v_2 - v_1)/(v_2 + v_2) = c',$$

then (25) becomes

$$\text{deflecting force} = c'/(A''p + B''' + 1/p). \quad \quad (27)$$

There is no need to take account of molecular force in altering density at edge of plate because so small a fraction of the free path lies in the condensed gas.
The last point to be attended to in applying our equations to the experimental results is that when one side of the mica vane is irradiated the glass bulb is also warmed in such a manner that it is hottest where nearest the candle, and therefore there is thermal transpiration along the inner surface of the bulb tending to raise the pressure near the hottest point with diminution towards the coldest point; now we can afford to neglect the effect of this near the vane until the pressure gets so small that the mean free path of a molecule becomes, say, nearly equal to the radius of the bulb, for then the walls of the bulb, on account of their much greater area than that of the effective edge of the vane, must dominate the distribution of temperature and pressure in the gas even quite close to the vane, and therefore at the highest exhaustions the relation between pressure and deflecting force must tend to a limit determined rather by the bulb than by the vane. With these explanations (27) is now applicable to the experiments of Crookes.

With his apparatus Crookes was able to study concurrently the viscosity of a gas and the forces at play in the radiometer at pressures from one atmosphere down to the lowest measurable by the McLeod gauge. The form of his vibrating system renders the mathematical problem of obtaining an expression for the viscosity of the gas from the constants of the apparatus and the observed decrement per vibration of the logarithm of the amplitude of the vibrations of the mica plate intractable; but it is obvious, from the theory of the vibrating disk method of measuring viscosity, that the motion of the mica plate when oscillating must be retarded by the viscosity of the gas in such a way that the difference of the logarithms of successive amplitudes is proportional to the viscosity of the gas, so that although absolute values of viscosity are unobtainable with the apparatus, approximate relative ones can be got with it. At a number of different densities of the gas Crookes measured the logarithmic decrement and also the repulsive effect of a candle-flame radiating towards the blackened half of the mica plate from a horizontal distance of half a metre, the latter being measured by a reading of the permanent deflexion of the plate from its position of rest in darkness.

Now from Maxwell's well known discovery that the viscosity of a gas is independent of its pressure it follows that the logarithmic decrement is independent of the pressure so long as slipping of the gas on the solid surfaces is negligible; but, as already indicated, Kundt and Warburg showed experimentally, with some support from theoretical reasoning, that slipping ceases to be negligible when the mean free path.

of the gas becomes comparable with the linear dimensions of the containing vessels; they did this by pushing the rarefaction of the gas so high in a vibrating-disk apparatus for measuring viscosity that the logarithmic decrement diminished measurably—for example, with air and a distance of 11 cm. between the fixed and moving plates the log. dec. at 1 atmo was 132, at 01 atmo it was 129, and at 0008 atmo it was 111; now at 20° C. and at these pressures the mean free path in air is about 00001 cm., 001 cm., and 012 cm. respectively, this last value is nearly 1/10 of the distance between the plates, so that when the distance between the plates is only 10 free paths the log. dec. diminishes by 16 per cent. of its limiting value when the distance is a large number of free paths. Thus we see how the measurements made by Crookes of the log. dec. in his apparatus give valuable information as to the relation between the free path of the gas and the distance from the edge of his mica plate to the glass bulb. In the following table the first row contains the pressures of dry air at 15° C. in terms of the atmo as unit, the second gives 10^4 times the log. dec., the limiting value of which at higher pressures is 1000, and the third contains the deflecting force of the candle in an arbitrary unit:

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<th>72</th>
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</tr>
</thead>
<tbody>
<tr>
<td>10^4 log. dec.</td>
<td>975</td>
<td>966</td>
<td>952</td>
<td>876</td>
<td>824</td>
<td>710</td>
</tr>
<tr>
<td>def. for.</td>
<td>exp.</td>
<td>3.5</td>
<td>5.5</td>
<td>10.0</td>
<td>27.0</td>
<td>32.9</td>
</tr>
<tr>
<td>cal.</td>
<td>4.0</td>
<td>5.9</td>
<td>9.6</td>
<td>25.0</td>
<td>31.3</td>
<td>39.7</td>
</tr>
<tr>
<td>press</td>
<td>36</td>
<td>29</td>
<td>19</td>
<td>13</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>10^4 log. dec.</td>
<td>695</td>
<td>657</td>
<td>577</td>
<td>500</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td>def. for.</td>
<td>exp.</td>
<td>42.5</td>
<td>42.6</td>
<td>38.8</td>
<td>30.9</td>
<td>27.1</td>
</tr>
<tr>
<td>cal.</td>
<td>40.7</td>
<td>40.8</td>
<td>36.8</td>
<td>30.0</td>
<td>26.8</td>
<td></td>
</tr>
</tbody>
</table>

Thus at a pressure between 36 and 29 millionths of an atmo the repulsion rises to a maximum, say at 30 millionths, at which the mean free path is 00001 x 10^6/30, that is one-third of a centimetre. The actual distance between the edge of the mica plate and the bulb is not given by Crookes, but from the figure he gives one would imagine that the distance might be between 5 cm. and 1 cm., and thus the experimental result corresponds to our theoretical one that the maximum effect is to be expected when D = 2\lambda; before the maximum is reached the repulsive effect ought, according to (27), to vary inversely as the pressure, so that the products of the numbers in the first and third rows above ought to be approximately constant, and the first four products are 2600,
2700, 3000, and 2700, which verify the equation. Beyond the maximum, according to (27), the repulsion is ultimately to vary directly as the pressure, so that the numbers in the third row divided by those in the first are to tend towards constancy: the last three values are 2:0, 2:4, and 2:5, while at lower pressures the value 3 is reached; but the results at these lower pressures have not been reproduced in the last table, because the M'Leod gauge with air becomes less reliable towards \( \frac{1}{10^6} \) atmo, and therefore Crookes's results at the lowest pressures will be discussed in a separate paper on the measurement of pressures in the highest attainable vacua.

From Crookes's experiments we can calculate \( c', \Delta'', B''/\sqrt{\Delta''} \) in (27), for with \( \frac{1}{10^6} \) atmo as unit of pressure and Crookes's arbitrary unit as the unit of repulsion, we have just seen that \( c'/\Delta'' \) is about 3000 and \( c' \) about 3:0, so that \( \Delta'' = .001 \); now the deflecting force is a maximum when \( p^2 = 1/\Delta'' \), so that the maximum value of the deflecting force, namely,

\[
c'/(2 + B''/\sqrt{\Delta''}),
\]
gives a convenient method of finding \( B''/\sqrt{\Delta''} \) when \( \Delta'' \) and \( c' \) are known; thus for air, \( B'' = .01 \) and we have all the data for calculating the deflecting force at any pressure by (27) for comparison with experiment: the calculated values are given in the fourth row of the last table, and show that we have the correct form of equation to represent the experimental facts. But according to the meanings of \( \Delta'' \) and \( B' \), \( B''/16 \) should be nearly equal to \( \Delta'' \), whereas \( B''/2/16 \) is only the \( 1/160 \) part of it.

Now the term in \( B''/\sqrt{\Delta''} \) expresses the effect of slipping, and our results for air show that in Crookes's apparatus the effect of slipping is only \( 1/160^4 \) or \( 1/13 \) of what it would be under the ideal conditions for (25), indeed (25) with the given values of \( A' \) and \( B' \) stands for one limiting case, and with \( B' = 0 \) it stands for the other where slipping is of no account, and the conditions of Crookes's experiments are nearer to those of the latter limit than of the former; indeed, with slightly different values of \( c' \) and \( \Delta'' \) we could put \( B''/\sqrt{\Delta''} = 0 \) and get nearly as good a representation of the experimental results for air as that just obtained. For nitrogen the values of the repulsive force are about two-thirds of those for air at the same pressures, except in the case of the small values, which are somewhat unreliable, thus for nitrogen \( \Delta'' \) and \( B''/\sqrt{\Delta''} \) have about the same values as for air, while \( c' \) is about 2:0; now according to equation (27) \( c' \), as it depends only on the dimensions of the apparatus and the temperatures of the two
Mr. W. Sutherland on Thermal

faces of the mica vane and conductivity, ought to have nearly the same value for two gases so closely alike as air and nitrogen; that is of course on the assumption that the value of $c'$ for oxygen is not much different from that for nitrogen, but we had better delay the discussion of this curious point until we have considered the data for oxygen.

For CO$_2$ and CO the parameters are:

<table>
<thead>
<tr>
<th></th>
<th>$c'$</th>
<th>$A''$</th>
<th>$B''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO$_2$</td>
<td>1.25</td>
<td>$\cdot000625$</td>
<td>0.0</td>
</tr>
<tr>
<td>CO</td>
<td>1.32</td>
<td>$\cdot000625$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

In the case of CO these values give values of the repulsion or deflecting force agreeing closely with the experimental over the whole range of pressure, but for CO$_2$ the calculated values are larger than the experimental at the higher pressures; but the matter is hardly worth going into more closely, especially as oxygen and hydrogen show exceptional behaviour of the highest interest to which we will proceed.

For oxygen Crookes obtained the following, the pressure unit being 1/10$^6$ atm, and the unit of repulsion the same arbitrary one as before:

<table>
<thead>
<tr>
<th></th>
<th>$p$</th>
<th>$10^4 \log_{10} \text{ dec.}$</th>
<th>def. force</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO</td>
<td>1000</td>
<td>803 658 623 613</td>
<td>12 12 13 13</td>
</tr>
<tr>
<td>CO</td>
<td>360</td>
<td>297 190 171 110</td>
<td>14 20 21 31</td>
</tr>
</tbody>
</table>

where the deflecting force remains almost constant from a pressure of 1000 down to 297, after which it rises, and at lower pressures than those given attains a maximum and then diminishes. Now Bohr (Wied. Ann. xxvii.) discovered a remarkable discontinuity in the compressibility of oxygen at about 921/10$^6$ atm, which has been corroborated by Baly and Ramsay (Phil. Mag. [5] xxxviii.), and obviously the above anomaly must be traced to the same cause as the discontinuity. These phenomena are so important for the chemistry of oxygen that I will discuss them in a separate paper on "Spontaneous Change of Oxygen into Ozone, and a remarkable type of Dissociation." Meanwhile we will go on to the region of pressure in which the repulsion in oxygen is not exceptional; here we have $c'=3.0$, $A''=0.007$, and $B''=0.132$, which give the following comparison with the experimental results:
Transpiration and Radiometer Motion.

<table>
<thead>
<tr>
<th>p</th>
<th>297</th>
<th>190</th>
<th>171</th>
<th>110</th>
<th>70</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>def. force.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cal.</td>
<td>13</td>
<td>20</td>
<td>22</td>
<td>30</td>
<td>40</td>
<td>44</td>
</tr>
<tr>
<td>exp.</td>
<td>14</td>
<td>20</td>
<td>21</td>
<td>31</td>
<td>38</td>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p</th>
<th>31</th>
<th>28</th>
<th>22</th>
<th>16</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>def. force.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cal.</td>
<td>45</td>
<td>44</td>
<td>40</td>
<td>34</td>
<td>29</td>
</tr>
<tr>
<td>exp.</td>
<td>44</td>
<td>44</td>
<td>40</td>
<td>35</td>
<td>30</td>
</tr>
</tbody>
</table>

The perfection of the agreement here emphasises the distinctness of the exceptional behaviour at the higher pressures.

In the case of hydrogen we have at the higher pressures the following values, the numbers in the third row being the products of repulsion by pressure, which ought according to (27) to be tending to a fixed limit:

<table>
<thead>
<tr>
<th>p</th>
<th>1000</th>
<th>921</th>
<th>526</th>
<th>421</th>
<th>330</th>
</tr>
</thead>
<tbody>
<tr>
<td>def. force.</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>921</td>
<td>1578</td>
<td>1684</td>
<td>1659</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p</th>
<th>314</th>
<th>234</th>
<th>205</th>
<th>179</th>
</tr>
</thead>
<tbody>
<tr>
<td>def. force.</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>2512</td>
<td>2574</td>
<td>2870</td>
<td>3222</td>
</tr>
</tbody>
</table>

The products do not show the same approach to a limit as was the case with air, and there is a jerkiness in their variation which points probably to experimental uncertainty. At the lower pressures the quotients of repulsion by pressure, which ought to be tending to a limit, are given in the third row of the following:

<table>
<thead>
<tr>
<th>p</th>
<th>16</th>
<th>14\5</th>
<th>12</th>
<th>8</th>
<th>6\5</th>
<th>5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>def. force.</td>
<td>52</td>
<td>49</td>
<td>45</td>
<td>37</td>
<td>31</td>
<td>29</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>3\2</td>
<td>3\4</td>
<td>3\7</td>
<td>4\6</td>
<td>4\8</td>
<td>5\3</td>
<td>6\5</td>
</tr>
</tbody>
</table>

Here again the convergence to a limit is not satisfactory, a state of affairs which will be traced in the separate paper on measurement of low pressures to inaccurate values of the lower pressures; and in that paper it will be shown that hydrogen exhibits a peculiarity which expresses itself in our equation (27) by dividing $A''$ by $(1-\alpha p)$ where $\alpha$ is another parameter, thus for hydrogen the deflecting force is

$$c'/\{A''p/(1-\alpha p)+B''+1/p\}, \quad \ldots (28)$$

which makes the deflecting force 0 when $p=1/\alpha$, a result to be extended to all higher pressures; the values of the parameters are $A''=0.0006, \quad \alpha=0.016, \quad B''=0.01$, and $c'=4.16$, which give the following comparison:

---
Mr. W. Sutherland on Thermal

<table>
<thead>
<tr>
<th>$p$</th>
<th>1000</th>
<th>526</th>
<th>330</th>
<th>314</th>
<th>205</th>
<th>147</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>def. force</td>
<td>cal.</td>
<td>0</td>
<td>2</td>
<td>10</td>
<td>11</td>
<td>21</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>exp.</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>14</td>
<td>28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p$</th>
<th>59</th>
<th>41</th>
<th>26.5</th>
<th>20</th>
<th>12</th>
<th>8</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>def. force</td>
<td>cal.</td>
<td>63</td>
<td>70</td>
<td>64</td>
<td>58</td>
<td>42</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>exp.</td>
<td>64</td>
<td>70</td>
<td>66</td>
<td>58</td>
<td>45</td>
<td>37</td>
</tr>
</tbody>
</table>

In view of the experimental uncertainty already pointed out at the high pressures and that which is to be proved at the lowest pressures, this comparison shows that the modified formula represents the facts for hydrogen about as well as possible.

We will now compare the theoretical values of the parameters $c'$ and $A''$ with the numerical ones just obtained; $c'$ stands for $bK(v_2 - v_1)/(v_2 + v_1)$, in which $K$ is proportional to $D$ and is a function of $k'$; also $(v_2 - v_1)/(v_2 + v_1)$ is the same for all the gases, so that $c'$ is proportional to $D$. But $A'' = 9D^2v_2^2/16\eta^2(v_2 + v_1)^2$, and $v$ varies as $m^{-1}$, and therefore $(A''\eta^2/m)^k$ is proportional to $D$, which now means distance from edge of vane to which transpiration extends. The following table contains $10^7k'$, $c'$, and $(A''\eta^2/m)^k$ for comparison; $\eta$ in terms of that for $O_2$ as 1, and $m$ in terms of that for $H_2$ as 2 are appended:

<table>
<thead>
<tr>
<th>$H_2$</th>
<th>Air</th>
<th>$O_2$</th>
<th>$N_2$</th>
<th>CO</th>
<th>CO$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^7k'$</td>
<td>3324</td>
<td>480</td>
<td>563</td>
<td>524</td>
<td>510</td>
</tr>
<tr>
<td>$c'$</td>
<td>4.16</td>
<td>3.0</td>
<td>3.0</td>
<td>2.0</td>
<td>1.32</td>
</tr>
<tr>
<td>$(A''\eta^2/m)^k$</td>
<td>7.62</td>
<td>5.20</td>
<td>4.68</td>
<td>5.20</td>
<td>4.11</td>
</tr>
<tr>
<td>$\eta$</td>
<td>44</td>
<td>90</td>
<td>1.0</td>
<td>87</td>
<td>87</td>
</tr>
<tr>
<td>$m$</td>
<td>2</td>
<td>28.8</td>
<td>32</td>
<td>28</td>
<td>28</td>
</tr>
</tbody>
</table>

It can be seen that on the whole these numbers confirm the theoretical conclusion that thermal transpiration near the edge of the plate is effective to a distance which increases with the conductivity. The smallness of $c'$ for $N_2$ as compared to its value for air and the smallness of $c'$ for CO are points that require confirmation by experiments with an apparatus lending itself better to quantitative calculations than the torsion radiometer of Crookes.

The values of $B'''$, as they represent only a small amount of slipping, and are not given very definitely by the experiments, are not worth further consideration.

So much for what may be called the static form of the radiometer; of the results obtained from a great variety of moving radiometers constructed by Crookes the following are the most important. In a radiometer containing two flies, one pivoted above the other, and having their blackened
sides facing in opposite directions, the radiation from a candle causes the flies to revolve in opposite directions, which proves that the driving action is chiefly localised close to the flies; this result is of course involved in our theory according to which the action of the fall of temperature through each vane is to raise the pressure near its hot face and lower it near the cool face, but the region of lower pressure of the upper fly being just above the region of higher pressure of the lower fly, and, with no obstruction between, ought to produce dissipation of the driving power of both flies, so that although they move in opposite directions they hinder one another in this direct manner as well as through the viscosity of the gas. Another radiometer contained only one pair of vanes at the ends of a single arm, and each vane carried opposite to its black face, at a distance of a millimetre, a large disk of thin clear mica; the action of a candle on this was to cause rotation in a direction opposite to the usual, that is, the black face moved towards the light. When another disk of thin clear mica was attached opposite the other side of each vane a candle ceased to have any effect. The theoretical reason for these facts is clear; in the first case the region of high pressure set up near the edge of the black face of the vane has more effect on the clear plate than on the vane and in the opposite direction, so that there is a resultant differential pressure driving the vane and its attachments in the opposite direction to the usual; when the other clear plate is attached there is an equal opposite resultant differential pressure due to it and so there is equilibrium; in short, when the two clear plates are attached the whole action is confined to the space between them, so that there can be no motion of the whole system.

In another radiometer the four vanes were left clear, but at the side of the bulb a plate of mica blackened on one side was fastened in a vertical plane passing through the centre of the bulb, so that a vane in passing it would leave a clear space of a millimetre; when light is thrown only on the clear vanes there is no motion, but as soon as it is allowed to fall on the fixed plate the fly revolves as though a wind were blowing from the black surface. This follows from theory at once, as the edge of the black face becomes a region of higher pressure and therefore a source of wind.

On replacing the pith or mica vanes by metallic ones Crookes encountered some new phenomena; perfectly flat aluminium vanes were found to be much less sensitive to the light of a candle than mica or pith; they move in the same direction, that is with the black surface away from the light, but when the candle is replaced by a source of dark heat their
Thermal Transpiration and Radiometer Motion.

motion is reversed, which is not the case with mica and pith (of course we are speaking of forms in which two or more vanes are arranged symmetrically with regard to the pivot); this reversal simply shows that the metal is a better absorber of dark heat than the lampblack.

But in working with vanes made of gold-leaf Crookes noticed that while the blackened side of one vane appeared to be repelled by a candle, that of another appeared to be attracted, and on examination it turned out that while the former vane was flat the latter was crumpled and bent in such a manner as to present a concave surface to the light. Following up this clue by constructing radiometers with bent and curved vanes Crookes was able to prove that in radiometer motion shape of the vane exercises even more influence than the absorbing power of the surface, so that a convex bright surface appears to be strongly repelled by a source of light, while a black surface if made concave to the light is actually attracted by it.

The theory of curved vanes is simple: consider a convex vane irradiated by a source on the normal through its middle point; then, as the amount of heat that a surface absorbs depends on its obliquity to the incident radiation, the farther a part of the convex surface is from the middle the less is it directly heated, and thus there is a continuous fall of temperature from the centre of the surface to the edge; conduction, if allowed time, tends to reduce the amount of the fall but does not obliterate it, and conduction also establishes a fall of temperature along the back from centre to edge; now the traction of the gas on the solid is from hot to cold, so that both on the front and the back of the vane there is a traction from centre to edge whose resultant effect is to drag the vane away from the light when the vane is convex to it, so that the light appears to repel a convex surface; when the surface is concave the same reasoning applies, the gas exerts a traction from centre to edge, and therefore the light appears to attract it.

There is hardly any need to reproduce any more of Crookes's facts or Pringsheim's skilful experimental analysis of the parts played in radiometer motion by bulb, vane, and gas; enough has been given to show that the kinetic theory can account qualitatively and quantitatively for all the essential facts of radiometer motion and furnishes general principles for the design of apparatus of the radiometer type. An illustration of the application of these principles will be given in a separate paper on "Two New Pressure-Gauges for the Highest Vacua."

Melbourne, August 1896.